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Importance of multiple time step optimization in river basin planning and management: a case study of Damodar River basin in India

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ABSTRACT

This paper outlines the importance of multiple time step optimization (MTO) in river basin allocation. The principal novelty of the work presented here is to provide a methodology for how to use MTO solutions in river basin planning and real-time operation. Two approaches for using the MTO results were tested on Damodar River basin in India and are presented in the paper. Using the proposed approach, the model managed flood flows without exceeding the downstream full bank channel capacity in 35 years of available historical data, while at the same time increasing generated hydropower on average by 63% annually, and supplying an additional 350 million m³ to irrigation and industry compared to the historical levels. The results presented in this study were obtained using the new Web-based Basin Management (WEB.BM) water management model, the only water allocation model with full Linear Programming (LP) optimization capabilities available online free of charge.

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Introduction

The use of computer models has become essential in river basin planning and management. A large number of computer models are used in the water resources sector, which differ in many ways depending on their purpose. This paper deals with river basin management models, which can be defined as models that find the most suitable set of reservoir releases and water use patterns over a specified time horizon. Several modelling approaches have been used in the past, generally divided into “rule based” and “optimization based,” where optimization models can be further subdivided into additional categories depending on the nature of the optimization engine (e.g. heuristic solvers or solvers based on some form of mathematical programming). Regardless of the actual modelling approach, most model vendors advertise the suitability of their models for addressing river basin management tasks. This is facilitated by the lack of strict acceptance criteria among water resources practitioners. In this context, the usefulness of the word “optimization” has been diminished, since everyone seems to claim that their model possesses the ability to “optimize” the search for the best solution, including the vendors of rule-based models such as MIKE-BASIN (Danish Hydraulic Institute 2020). Recent initiatives related to the model selection process for Narmada River basin in India were aimed at introducing more technical rigour into this field, thus setting higher expectations for model vendors regarding future applications of their models (Ilich *et al.* 2019).

Usually, decision-making processes in water resource models either are based on a set of user-specified operating rules, or use some type of mathematical optimization approach aimed to find the best set of hypothetical regulated flows that

minimizes or maximizes a given objective function. These models simulate decisions of reservoir operators and river basin managers. For example, in times of water shortage, these models should be able to provide solutions that can bypass upstream users and provide water to downstream users that have higher priorities.

To demonstrate the wide variety of existing models, reservoir operation models were briefly reviewed by Wurbs (1993), and later by Labadie (2004). These two papers present a compressed summary of more than 50 models. There is no single favourite river basin management model with widespread use among practitioners. While some model vendors have invested considerable effort in advertising and promoting their models, a model’s capabilities and performance can only be demonstrated by using it to provide successful solutions to challenging test problems – demonstrations which are sorely lacking in the relevant literature. The only test problem that has repeatedly been used in the industry dates back to 1979 (Murray and Yakowitz 1979), with four reservoirs and 12 consecutive time steps. Both this problem and its larger version (with 10 reservoirs over 48 monthly time steps) are very simple. They exclude net evaporation on reservoirs and variable outflow capacities dictated by the available storage, and they were mainly used to test new solution strategies that rely on various heuristic solvers.

Early optimization-based river basin management models for water distribution along water resource networks used simplified linear programming algorithms, specifically designed for network flows, typically referred to as network flow algorithms (NFAs). While NFAs offer high solution speed, they are unable to include dynamic flow constraints that are typical for water resource networks. The only constraints that can be modelled within the NFA framework are

the continuity of flow at each node and the upper and lower flow limits in flow links such as canals. Constraints such as the return flows from irrigation districts that depend on the simulated water use, or maximum reservoir releases that depend on average storage over a simulated time step, were initially addressed within NFA solvers with iterations. Unfortunately, iterations often fail to converge to optimal solutions, and may instead converge in the wrong direction (Ilich 2009).

Despite this, many NFA-based models such as MODSIM (Colorado State University 2020), RESOURCE ALLOCATION MODEL (REALM) (Gov. of Victoria 2020) and Hydrologic Engineering Centre Reservoir Prescriptive Management (HEC-ResPRM) (US Corp of Engineers 2019) became popular among practitioners over the years. The shortcomings of the NFA models were noted, with eventual use of commercial mixed-integer Linear Programming (LP) solvers that were incorporated in models such as RiverWare (Zagona *et al.* 2001), OASIS (Rundall *et al.* 1997), Hydrologic Engineering Center - Flood Control Linear Programming (HEC-FCLP) (Needham *et al.* 2000), and Web-based Basin Management (WEB.BM) (Ilich 2019). Mixed-integer programming (MIP) solvers were needed instead due to the use of binary variables required to ensure the right sequence of filling and emptying storage zones on reservoirs (Ilich 2008).

River basin allocation models have traditionally been run by simulating a sequence of individual decisions made in each simulated time step, which is usually referred to as single time step optimization (STO). In such runs, an optimization engine is still applicable for allocating water among competing stakeholders; however, a decision related to water release in the current time step affects available management options in subsequent time steps, so to truly optimize water allocation for an entire irrigation season or a hydrological year, the model should solve a sequence of several consecutive time steps assuming known runoff forecasts and water demands. This is known as multiple time step optimization (MTO). The reasons the MTO modelling approach is rarely used among practitioners are: (a) very few models are capable of delivering these solutions and they are rather expensive (e.g. RiverWare and OASIS); (b) the MTO model computation times are much longer and their debugging requires above-average technical skills; and, most importantly, (c) there are no clear guidelines on how to use the model outputs, based on the fact that long-term hydrological forecasts are not available.

The principal novelty and contribution of this paper is to show that there is an effective way to both develop and interpret MTO solutions such that they can (a) help revise the existing operating rules; and (b) use the MTO as a real-time reservoir operational tool over a short time horizon, based on combining the revised operating rules developed under (a) in a combination with short-term runoff forecast models. The paper demonstrates the potential value of this approach on a case study in India.

In the first section, the paper presents the origin of the rule curve concept and discusses its use to improve STO solutions, as well as its shortcomings. This is followed by a presentation of the MTO solution concept and its advantages over the STO solutions. The next section lays out the basis of the methodology developed in this study, followed by its application on a case study, and a brief review of an alternative attempt to address the problem of learning from MTO solutions by using the pattern-matching algorithm. The discussion section and

the concluding remarks summarize the findings of the two approaches that were investigated and provide suggestions for further research and development.

The concept of reservoir rule curves

A significant majority of popular water allocation models, such as MODSIM (Colorado State University 2020), REALM (Gov. of Victoria 2020), AQUATOOL (Haro *et al.* 2012) and HEC-ResSIM (US Army Corps of Engineers 2020) use STO solutions, which means that they do not take into account forthcoming hydrological conditions and water demands beyond the length of a single time step. The principal disadvantage of this approach is displayed in the two graphs at the bottom of Fig. 1, which show crop failure in both years. To avoid this, irrigation managers typically hedge their demands in extremely dry years – i.e. they lower their targets to reduce the chances of crop failure. Their dilemma is then to determine the level of reduction that is the most appropriate for the current conditions, which is currently a matter of the gut feeling of the operators and management committees.

The concept of the reservoir rule curve was developed by Ravelle and Kirby (1970) to avoid the issues, presented in Fig. 1, that are associated with the STO solution procedure. The rule curves are shown as dashed lines in the top left diagram in Fig. 1. This concept allows model users to define maximum permissible drawdown water levels, with a high penalty factor associated with their violation. In essence, the rule curve shown in Fig. 1 defines the amount of usable storage during an irrigation season, where the drop of storage levels in the curve from one time step to the next defines the maximum storage drawdown.

Most river basin planning models rely on reservoir operating rules that are defined by the shape of their rule curves. Rule curves are usually static target water levels that do not change from year to year. Yet their shape is a function of the storage at the start of an irrigation season, and of incoming runoff and estimated water demands throughout the irrigation season, which implies uniqueness in every year. The concept is not applicable in the case of two or more sequential dry years, which make it impossible for the model to follow the desired rule curve shape in the second and subsequent dry years due to insufficient starting elevation and runoff required for simultaneous refill of storage and downstream consumption. The idea of constructing the reservoir rule curve and using it as a model constraint can be understood as imposing a user-defined solution in the input data file, which defies the purpose of optimization. In reality, the best set of reservoir levels can only be developed as part of the optimized model output, unique for every simulated year, and also inclusive of the optimal managing of water demands (hedging) in dry years. Since the original concept was proposed by Ravelle in 1970, rule curve development has remained in the domain of the trial-and-error approach, with its ultimate shape being heavily dependent on the judgement of the modeler, rather than on the results of a standardized procedure or a widely accepted approach.

The main advantages of the multiple time step optimization (MTO) modelling approach presented in the next section are:

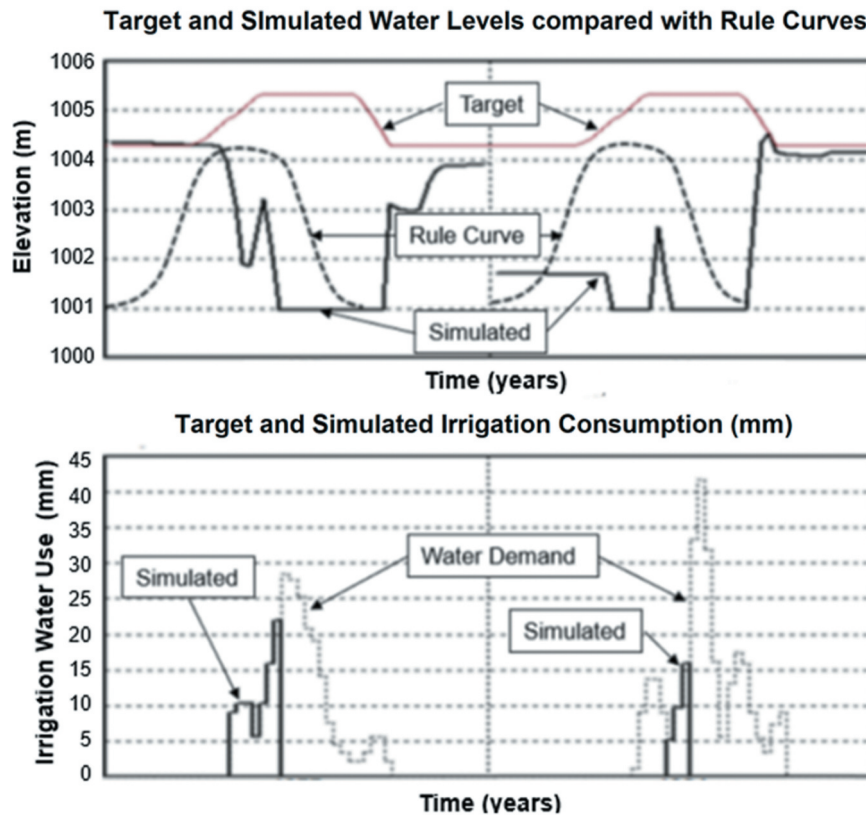


Figure 1. Reservoir levels (above) and achieved vs demanded supply (below) for STO.

- For planning studies, the model input does not need any rule curves; and,
- MTO results contain a perfect rule curve for each simulated year. As such, statistical analyses of the model results can provide insight into better operating rules, especially when they are based on a large number of simulated hydrological input sequences that can be produced using a reliable stochastic flow generation model.

In addition to the above, MTO solutions can combine perfect reservoir operation with optimized demand hedging policies, which are also obtained as part of the model solution via incorporating the appropriate constraints in the model.

Multiple time step optimization

MTO is based on the assumption that inflows for several consecutive forthcoming time steps are known. In planning scenarios, the entire historical inflow series is provided to the model as known inflows. Since water demands are also known, this allows the model to determine perfect reservoir levels and demand hedging policy in a single run for each year, without the need to resort to any iterative schemes. The MTO solution approach is not entirely new; it was used to compile the California State Water Plan (Lund 2003), but so far there has been no accepted methodology established on how to use long-term perfect model solutions to improve future reservoir operations in real time when inflows are not known.

As a demonstration of the MTO approach, the model set-up is shown for three consecutive time steps in Fig. 2. This example

has one diversion canal, one irrigation block, one reservoir and two river reaches. The use of only three time steps in Fig. 2 is aimed to explain the concept that can be extended to any number of time steps with available inflows and water demand data. Channel flows and ending storage volumes for each time step feature as decision variables, which are labelled as variables $X_{i,t}$ in Fig. 2, where the subscript i represents the modelled component while the subscript t represents the time interval. Variables can be either in units of volume or, as assumed in this case, in units of flow, so that $[V_{init}(1)/t]$ represents reservoir storage in the units of flow at the start of the simulation. For single time step solutions (STO mode), the schematic in Fig. 2 would only have the left third, showing the reservoir in the first week with inflow, diversion into an irrigation block and the outflow channel. When there is enough water supply, then $X_{i,t} = D_{i,b}$ implying that the deficits are equal to zero.

In the MTO solution mode, the same physical reservoir is connected with the storage carry-over arc, which ensures that the ending storage from one time step automatically begins the starting storage for the subsequent time step. Although net evaporation is not included in Fig. 2, it should be modelled as a gain or loss on the storage carry-over arc. It is necessary to define the value of P_i per unit of flow for each component shown in Fig. 2, including storage, which typically assumes the lowest value of P_i compared to other components in order to conserve excess runoff after all downstream demands are met. To ensure allocation, users must assume that supplies to irrigation block defined by variables $X_{i,t}$ have a higher value of P_i per unit of flow compared to its value assigned to storage. The weight factors assigned to flows in the river reaches are set to

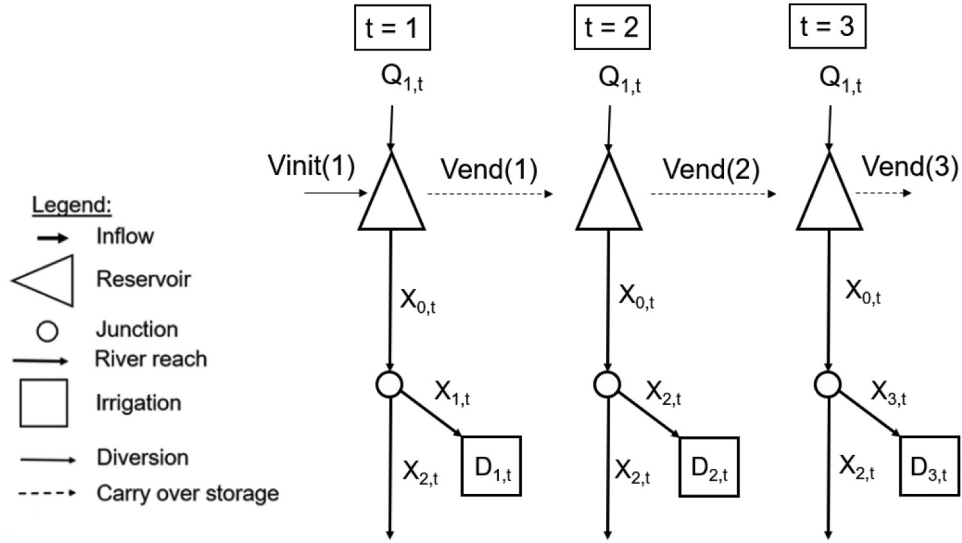


Figure 2. Sample MTO configuration for three consecutive time steps.

zero unless there are environmental flow targets to be maintained, in which case the weight factor would be assigned a positive value on the basis of its priority compared to other water users. We denote carry-over storage as a variable, since it enables balancing of storage between consecutive time intervals, i.e.

$$X_{4,t} = \frac{V_{1,t}}{\Delta t} \quad (1)$$

where Δt is the length of the calculation time step. The objective function is specified as:

$$\text{Max} \sum_{i=1}^n \sum_{t=1}^m X_{i,t} P_i \quad (2)$$

where in the above example $n = 5$ is the number of components, which include one reservoir, one irrigation block and three channels, while $m = 3$ is the number of time intervals shown in Fig. 2, although in planning studies m should be equal to the number of time intervals within a year. In its simplest application, the above objective function is subject to the following constraints:

$$\sum_{i=1}^n \sum_{t=1}^m (X_{i,t} + Q_{i,t}) = 0 \quad (3)$$

$$0 \leq X_{i,t} \leq U_{i,t} \quad (4)$$

Equation (3) represent the mass balance constraint, while Equation (4) sets the lower and upper bound on all variables. Inflows $Q_{i,t}$ are treated as given inputs of available runoffs at a node, and they can theoretically be given for any node in the network, although in practice only some of the nodes will have inflow series representing the available sub-catchment runoff. Upper bounds $U_{i,t}$ in Equation (4) represent limits associated with the storage or flow capacity of canals. Irrigation demands also have their targets set for each time step, which will be represented as the upper bounds. To ensure equal spread of deficits throughout an irrigation season in a dry year, the

following constraint can be added to equate the ratio of supplied and demanded quantities in all time intervals:

$$\frac{X_{1,t+1}}{D_{1,t+1}} = \frac{X_{1,t}}{D_{1,t}} \quad (5)$$

In dry years, when irrigation deficits are inevitable, the model finds the minimum deficit for the entire year by combining storage and the available runoff, and spreads it evenly throughout the year. This amounts to optimized demand hedging, which is conducted simultaneously with finding optimal reservoir operation in each year and for each reservoir in the system. Other constraints were added in this study to handle net evaporation and dynamic flow limits through the hydropower plants as a function of the available storage and the capacities of turbines and generators. Future model refinements of this study may include outflow limits on spillways as a function of the available storage, return flows that should be modelled as a fraction of consumptive use, or runoff apportionment agreements between bordering states once such agreements are contemplated as model inputs.

The way the optimization problem is defined in Equations (2) through (4) could also be solved with an NFA algorithm, although the required solver would have to include the ability of loss or gain of flow along an arc to properly model net evaporation. In addition to this, the best known NFA-based models such as MODSIM or REALM cannot handle MTO solution approaches at all; they can only solve STO allocations. Adding Equation (5) makes this problem ineligible for use with any NFA solution approach. Additional complexity that cannot be handled by NFA algorithms includes the reservoir outflow constraints, which need to be specified as a piece-wise linearized function of the available storage over a time step. The similar relationship between the maximum possible flow and the average storage over a time step also needs to be linearized for flows through hydropower plants.

In addition to all of the above, the model should be able to handle variable time step lengths, which in this study range from 10 days in the dry season to 3 days in the monsoon

season. Of all available linear programming-based models, only two are currently capable of handling all of the above requirements: OASIS and WEB.BM. Unlike OASIS, the WEB.BM model, which was used in this study, is web based and freely available online via www.optimal-solutions-ltd.com, or alternatively at www.riverbasinmanagement.com.

The last point to make related to the MTO solutions is the fact that they rely on perfect foreknowledge of inflow hydrographs for the entire season (or hydrological year), which is never available in practice. This raises a legitimate question: how should the results of MTO optimization be used? In their recent comprehensive article that includes an in-depth survey of the available literature on heuristic solvers, Dobson *et al.* (2019) review the use of optimization, distinguish between rule curve-based models and multiple time step solutions, and outline the need to apply artificial intelligence algorithms that can learn from numerous MTO solutions and apply their results in real-time operation.

To enlarge a learning database of MTO solutions, researchers typically resort to the use of implicit stochastic optimization, where lengthy stochastic flow frequencies are first developed and fed to the reservoir optimization models so as to produce a multitude of optimal solutions that serve as input into various inferential models with an aim to “guess” the best reservoir releases based on the current state of the system, such as the current storage levels, along with the recent or forecasted precipitation or inflows.

Willis *et al.* (1984) present typical work that outlines this approach, where the optimization results are regressed against the starting storage and monthly inflow forecast in each monthly solution, thus providing the monthly forecast of reservoir releases based on the month of the year and its starting storage. Virtually all publications that followed used a variation of the same theme, with non-linear regression used by, for example, or other inferential models such as neural networks (Chandramouli and Deka 2005).

Gavahi *et al.* (2019) provide an example of a single reservoir optimization problem solved with MTO where the solutions are fed as input into an adaptive neuro-fuzzy system along with 1 month inflow forecasts based on the regression of the flows in the 3 previous months. The intention was to predict optimal monthly releases in real time that are similar to the MTO solutions for similar inflow conditions and starting reservoir levels, without using any user defined rule curves. While the use of linear programming guarantees the best MTO solutions in this study, monthly time step solutions are not suitable as a guide for real-time operation, and most water resources systems nowadays have multiple reservoirs that need to be operated as a system, not as individual units.

Rani and Moreira (2010) provide an excellent summary of the state of the art of the currently available reservoir modeling tools. Most research publications in this area deal with monthly time steps and single reservoir systems, which explains to some extent the gap between the theory and practice, since (a) inflow forecasts are neither constant over a month nor available over the entire month in real time; and (b) modern water resources systems are typically multi-reservoir and multi-purpose.

The inherent problem with the use of inferential algorithms that are supposed to set the best reservoir outflows based on the state variables and the time of the year is that they are unable to accurately take into account downstream constraints that should not be violated, such as the full bank flow capacity or the exact amount of water demand on an irrigation block. Only an optimization model that takes into account the mass balance, water demands and constraints can take all of the above into account properly, and derive the best solution based on the use of mathematical programming.

This has been recognized in the past, and Yazicigil *et al.* (1983) is a rare example of published work that tackles the problem in the right way. This study derives daily reservoir releases on the basis of inflow forecast over a 5 days horizon, by solving MTO for 5 d ahead. There are multiple issues with the way the solution is derived. The storage outflow vs elevation constraints were not included in the model, nor is there a proper hydrological routing in sequential downstream river reaches, and it uses questionable rule curves to guide the model solutions that were taken from other studies based on simplistic simulation models. However, the idea that a short time horizon may provide a reliable flow forecast that could be used to drive real-time operation for 5 d ahead, where only the solution for the first day would be adopted for real-time operation, while this process is repeated for every subsequent day, is a good approach that has been adopted in this study. This idea is laid out in more detail in the following section, but the basics that outline the uniqueness of the approach are listed below. In short, there are two novelties in the proposed approach:

- (a) A unique way to construct the reservoir operating zones by using the MTO approach and by conducting statistical analyses of MTO solutions; and,
- (b) The use of MTO for real-time operation, on the basis of the operating zones constructed in phase (a) combined with the availability of short-term runoff forecasts.

In addition to the above, an attempt was also made to generate estimates of reservoir outflows based on a pattern-matching algorithm that matched the inflows and the starting storage with the median release obtained from filtering the eligible solutions from a database of 1000 years of MTO solutions based on stochastic inflow and precipitation series. The proposed methodology for both approaches and the results obtained from the case study are presented below.

Methodology

A block diagram representing the proposed approach is shown in Fig. 3. The stochastic generation of inflows may not be needed if the historical series is sufficiently long (i.e. close to 100 years). However, this is usually not the case, and in general a larger statistical sample provides more reliability in any statistical analysis.

Historical natural flows should be determined by a process that involves the removal of the effects of storage reservoirs and diversions caused by human intervention. This is achieved by recalculating reservoir inflows and by adding back the historical water abstractions to the recorded river flows. This

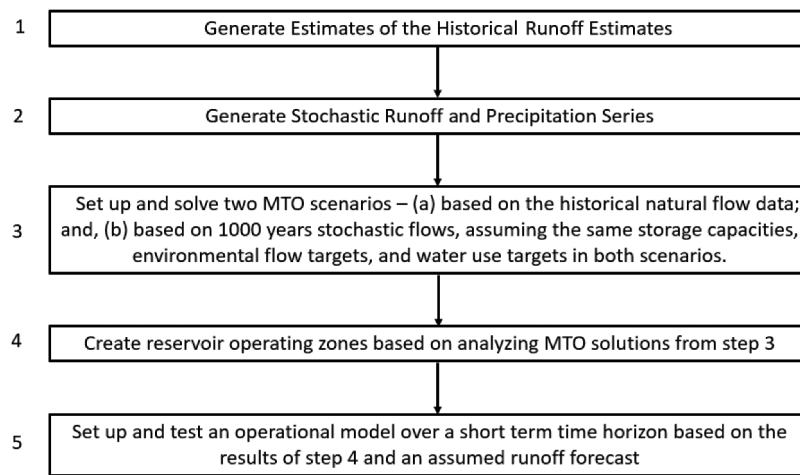


Figure 3. Block diagram of the proposed methodology.

is a much more reliable approach to estimate natural runoff than the use of rainfall–runoff models, which may be calibrated for a few years of rainfall and runoff data, but which often fail in the verification phase over longer time periods. Rainfall–runoff models should be used to estimate natural runoff only as a last resort option.

A stochastic model can extend the length of the hydrological input and generate many more combinations of back-to-back dry or wet years, the likes of which are not available in the historical records. The results of any stochastic generation of hydrological time series should be verified by comparing its statistics with the statistics of the historical series.

Step 3 refers to the MTO model set-up and preparation of all required input data generated in steps 1 and 2. The resulting output is a database of 1000 years of optimal solutions that contain reservoir levels and channel flows for each time step of each simulated year. Step 4 summarizes the process of creating reservoir operating zones on the basis of the MTO solutions.

Simulated storage levels follow a typical pattern of draw-down and refill in each simulated year. When derived using the

MTO solutions, they represent perfect reservoir operating rules for each simulated year. If they were all plotted on a single graph there would be too many lines, which would make visual inspection illegible. Instead, a probability density function can be constructed for the end of each time step, as displayed in Fig. 4. This strengthens the argument for using 1000 years of stochastic solutions rather than only 35 years of historical solutions, since a larger data sample results in a more reliable statistical function that covers a wider range of probabilities. It may be noted that studies of this kind can be done without stochastic hydrological series in instances where the historical records are close to 100 years long. However, this situation is rare for most river basins.

Assume that all median values for the relevant statistical distribution functions in each time step are connected. This should represent the most likely simulated water levels for the entire year. Selected elevations with given percentile probabilities (e.g. 10th, 20th, 80th or 90th percentiles) at the end of all simulated time steps can be connected in a similar fashion to create the operating zones for all five reservoirs based on the

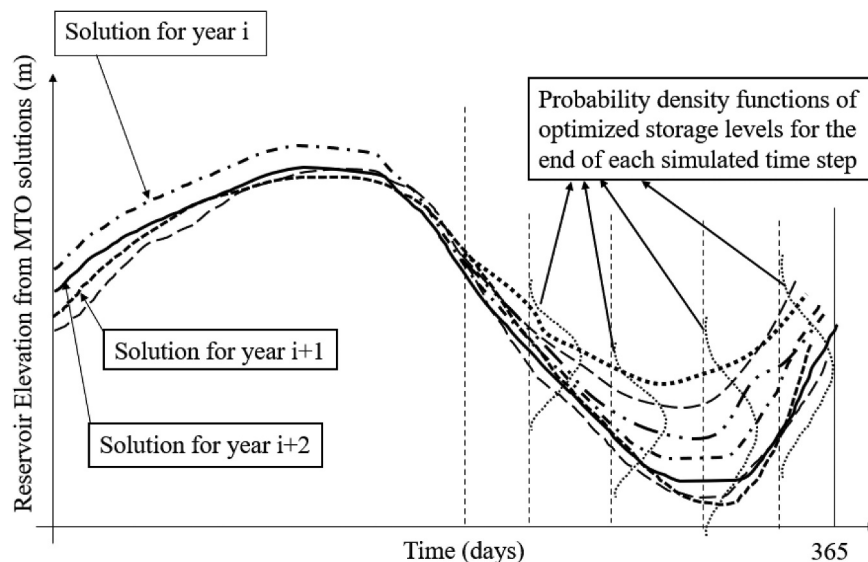


Figure 4. Statistical analyses of MTO solutions.

MTO solution results, thus creating an MTO-based design process of reservoir operating zones.

Step 5 involves a different model set-up for short-term operation that utilizes the zones created in Step 4. The inter-reservoir operating rules from step 4 determine the sequence of reservoir releases during the normal operation. The number of zones is the same for all reservoirs, and the assumed operating rule is that all reservoirs remain in the zone of the same order. During normal operation, zones are emptied and refilled in a sequential order throughout the year, and this policy merely helps the model determine the sequence of storage releases for downstream needs that can be met by a numerous combination of releases in multi-reservoir systems. The actual releases are made by a sequence of MTO solutions over a short time horizon for which the runoff forecasts are assumed to be available, and they may occasionally violate the storage sharing concepts in extraordinary circumstances when managing floods by using a combination of pre-flood drawdown and controlled releases that are derived as part of the MTO solution. An alternative approach that has been tested differs from the above in the last two steps shown in Fig. 5.

It can be noted that the alternative algorithm does not attempt to explicitly derive reservoir outflows, but rather attempts to guess the ending storage levels for the end of the current time step, since it is matching the current conditions with the database of MTO solutions realized over the stochastic inflow series. Given the inflow forecast and the ending storage, it is possible to retrieve the desired outflow by using the water balance calculation for each reservoir.

The following section describes the application of the above approaches to a real-world problem, and discusses the results of the numerical experiments that were conducted using each of the proposed methods.

Damodar River basin

Damodar River basin is located in West Bengal, India. There are five reservoirs in the basin, designed for multi-purpose water uses including water supply for irrigation, industries and municipalities, hydropower generation and flood protection in the downstream reaches, as shown in the modelling schematic in Fig. 6, with the squares representing irrigation water use. The basin is currently operated using previously developed static reservoir rule curves.

Panchet and Maithon are the largest dams in Damodar basin, with total installed hydropower plant capacities of 80 and 60 MW, respectively. The management challenge in this basin is to operate the five reservoirs in the system so as to

minimize flood damage in the most downstream river reach, while simultaneously maximizing hydropower production and maintaining required water supply for irrigation, municipalities and industries. Historically, water supply was manageable; however, water demands will be increased significantly in future, making the task of delivering required amounts to all users more challenging. The principal issue in this study is to manage floods and generated power, such that flood damage is reduced, with minimal negative overall impacts on hydropower producers. There are 35 years of historical hydrological data provided in this study, starting in 1981. A modelling time step duration from 10 d in the dry season to 3 d in the period 1 July to 30 September was set up, with a gradual transition in the months of June and October with time steps of 4 or 5 d. Modelling was based on running a continuous simulation with a variable time step length for all 35 historical years of record. Since 35 years is not a long period for statistical analyses of model output, an alternative stochastic 1000-year hydrological series of flows and precipitation was created and used as model input. The shortest time step of 3 d was based on the total travel time between Tilaiya and Maithon reservoir being slightly less than 2 d, which removed the need to use hydrological routing, for which there were no sufficient data for calibration at this point. Hence, the model was run on a steady-state basis. Shorter time steps introduced more challenge when modelling monsoon floods.

Description of modelling scenarios

The following scenarios were set up:

- (a) Verification scenario: historical reservoir outflows in this scenario were enforced, and the resulting simulated reservoir levels were compared with the historical levels. A perfect match between the historical and simulated reservoir levels confirmed correct historical runoff estimates. Note that a perfect match is only possible if runoff is calculated correctly, since there are no parameters that need to be calibrated.
- (b) Optimization scenario: the same runoff estimates confirmed in the verification scenario were used as inflows, and the same starting reservoir elevations as in 1981, but the model was set up to find the best reservoir operation that maximizes the stated objectives, which were to
 - minimize damage from flooding downstream of Durgapur Barrage (the last river reach shown in the modelling schematic in Fig. 3;

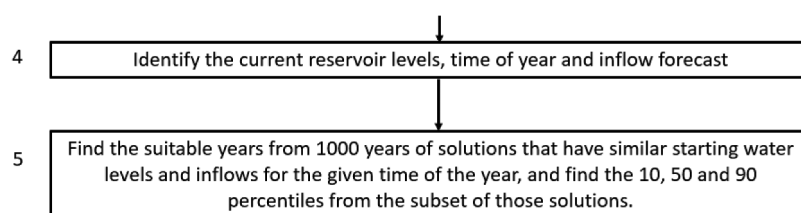


Figure 5. Alternative pattern-matching algorithm.



Figure 6. Damodar River basin modelling schematic.

- meet all municipal, industrial and irrigation demands; and
 - maximize generated hydropower.
- (c) Stochastic optimization scenario: This scenario used 1000 years of stochastically generated hydrological input data series of flows and precipitation. The purpose of this scenario was to analyse optimal reservoir levels and provide insight into alternative rule curves that could better address the management goals. The final output of this scenario is the adjusted reservoir operating zones for all reservoirs, with the target zone based on the 50th percentile elevations for each time step that were obtained from the 1000 years of optimal basin operation. Other statistics of simulated water levels, such as 10th and 90th percentiles, were also used in the definition of reservoir operating zones.
- (d) The final scenario included a combination of the adjusted rule curves and operating zones with the short-term model runs based on MTO solutions for two consecutive time steps. During the monsoon season, two consecutive time steps would require inflow forecasts for a total of 6 d.

To avoid downstream flooding, the model was constrained to keep the combined outflows from Maithon and Panchet to less than 3300 m³/s. Both historical and optimal scenarios included net evaporation losses on reservoirs as a function of the average reservoir area over a time step. Historical operation showed water levels in the flood storage zone (above 146.3 m

for Maithon and 124.97 m for Panchet) in many years in which there was no flooding, which was likely inspired by the desire to increase generated hydropower. Also, in many historical years reservoirs are drawn down much more than necessary, probably due to the policy to follow a similar operating rule curve in each year, which caused unnecessary loss of generated power in moderate and wet years. Figures 7 and 8 compare historical and optimized reservoir operation. The verification run (historical operation) and the results of two model runs are compared in more detail later.

Figure 9 shows the flows in the Damodar River below Durgapur Barrage. Optimized simulation was based on high maintenance flows of 184 m³/s from July to October, which are always achieved in optimal simulation, but which were not always achieved historically despite being a stated goal. Four historical violations of the 3300 m³/s flow limit were recorded in the period from 2000 to 2010, as shown in Fig. 9. Although there are large water demands downstream of both Maithon and Panchet Dams, they are all met without much drawdown on the Panchet reservoir. This can be explained by the modified operation of the Konar and Tenughat dams, located upstream of Panchet. The optimization algorithm forced releases from these two reservoirs first to maintain Panchet water levels close to normal and to provide sufficient outflows for all downstream water demands. This was driven in part by the objective to maximize generated hydropower, with the Panchet dam having the largest hydropower capacity (80 MW) in the system.

It becomes obvious that overall, considerable improvement in basin management is theoretically possible. However, the optimal solutions in Figs 7–8 are based on perfect foreknowledge of

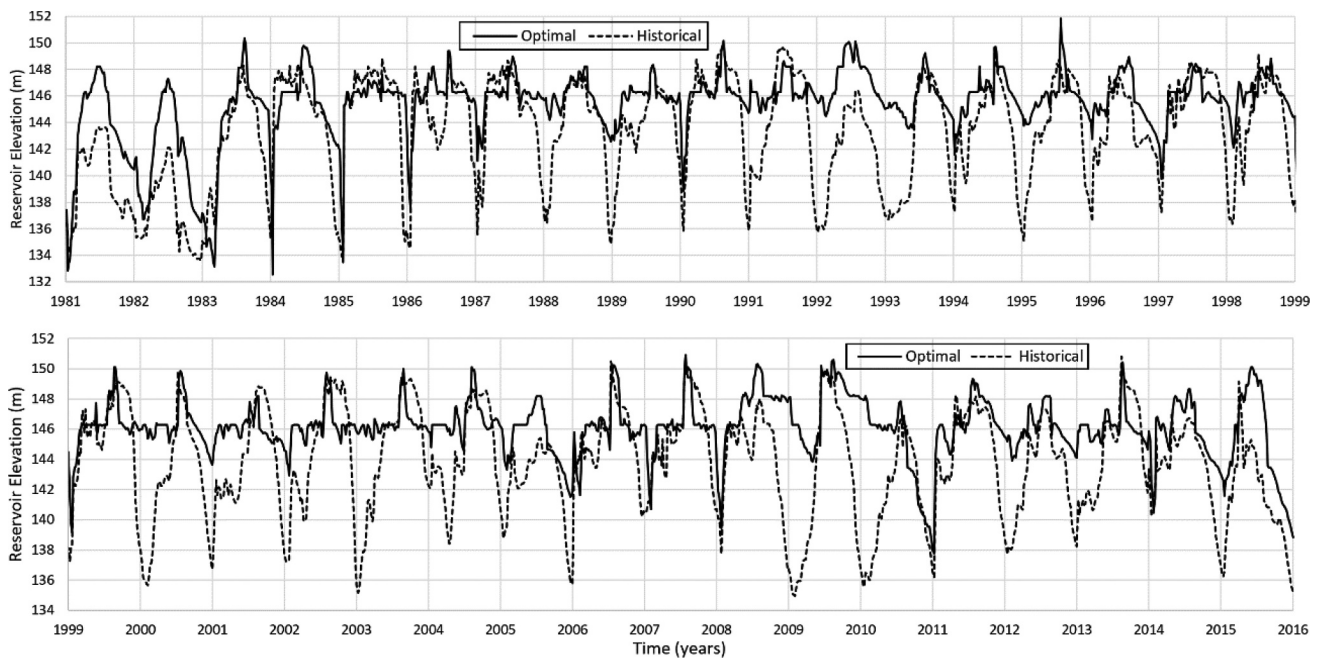


Figure 7. Comparison of historical and optimal Maithon reservoir operation.

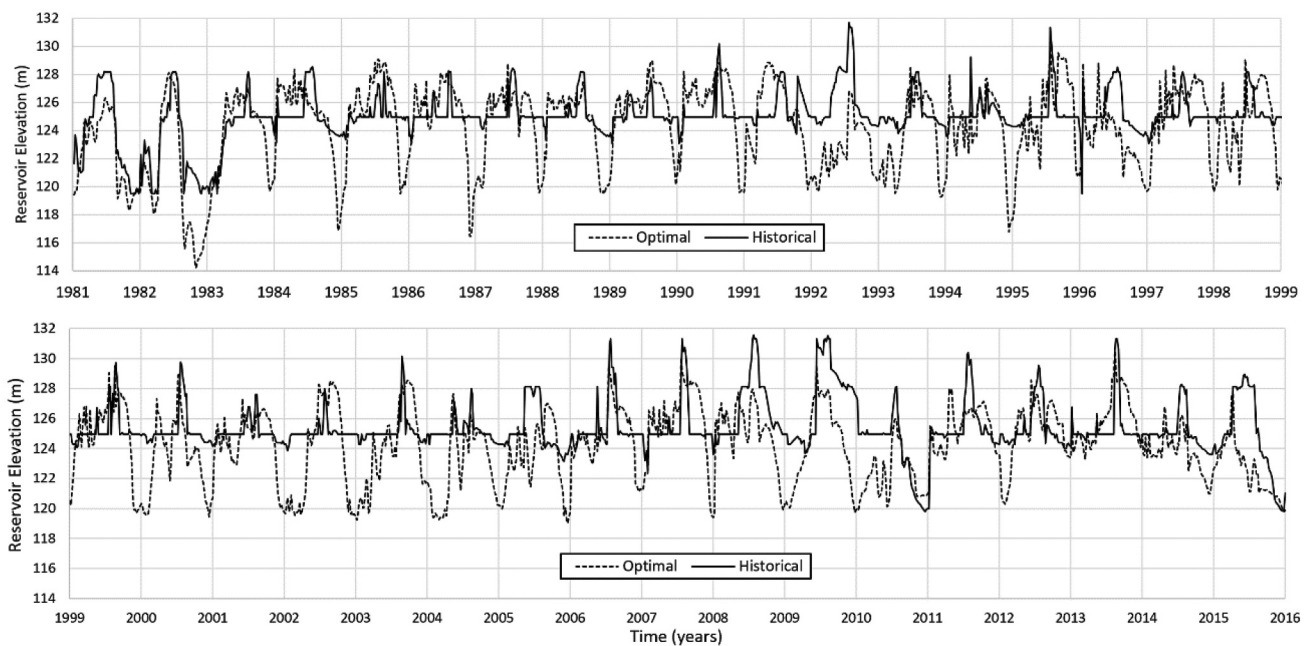


Figure 8. Comparison of historical and optimal Panchet reservoir operation.

inflows for the entire year. The pertinent question is: How do we use this information, especially since the historical series of 35 years will never repeat exactly the way it unfolded between 1981 and 2016? The first step is to statistically examine the output. Figures 10–12 show the difference between the historical and simulated 90th percentile, median and 10th percentile water levels for Maithon, Panchet and Tenughat dams.

Historical operation of Tenughat reservoir often involved water levels lower than the design. The reasons the reservoir

was operated this way historically might be that in the earlier days it was operated as an independent single reservoir under what was then Government of Bihar (which later came under Jharkhand after the rearrangement of the state of Bihar in November 2000). However, the model utilized the full live storage at the design level, and added additional storage at Konar dam for meeting all downstream demands, while keeping the water levels at Panchet Dam significantly higher than during historical operation, which consequently produced more hydropower. During

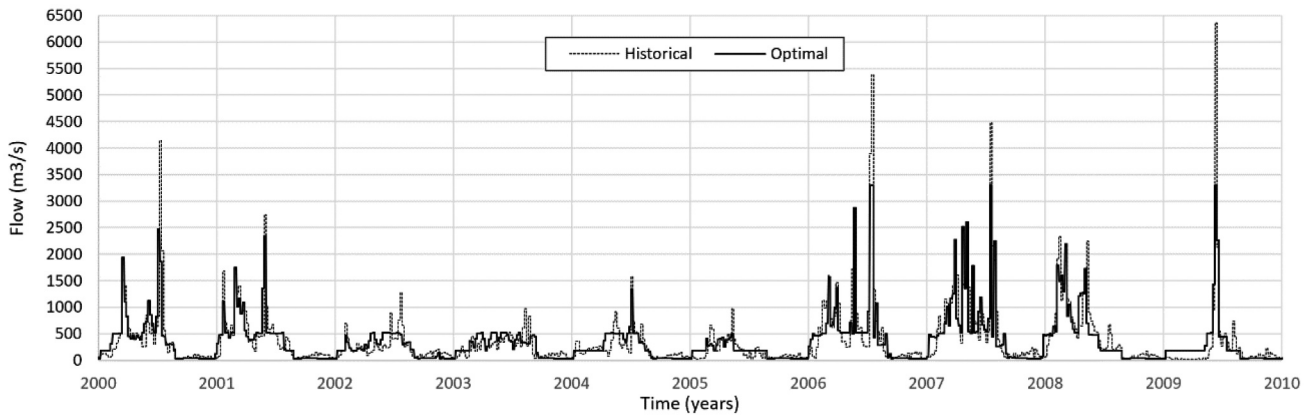


Figure 9. Comparison of historical and optimal flows below Durgapur Barrage.

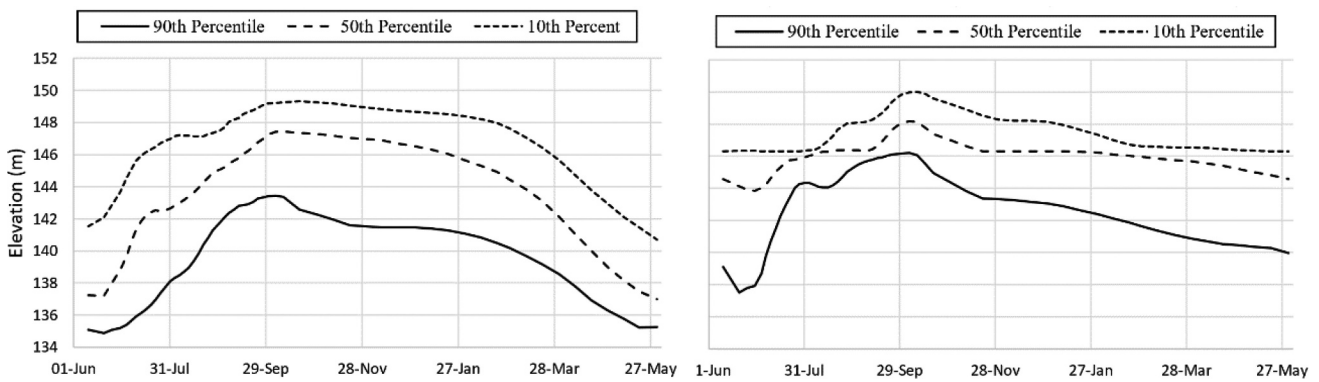


Figure 10. Maithon reservoir historical (left) and simulated (right) water level statistics.

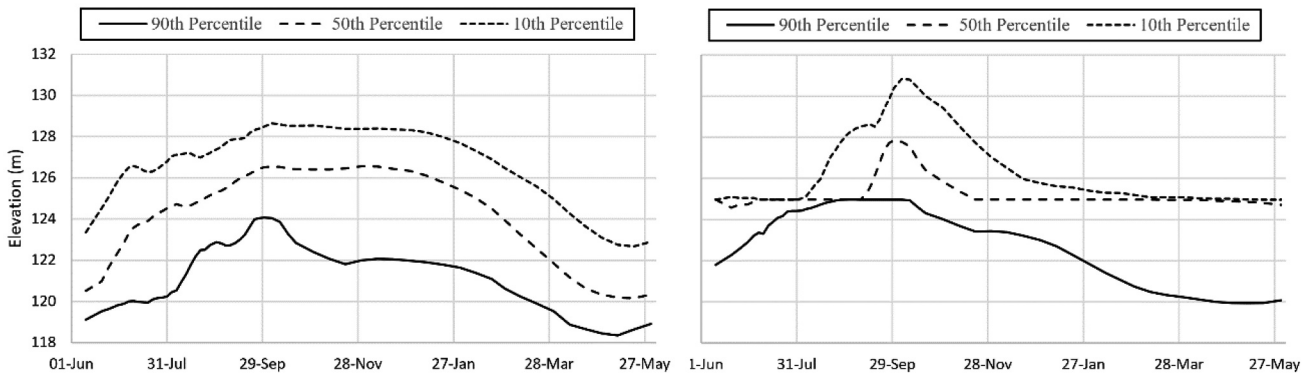


Figure 11. Panchet reservoir historical (left) and simulated (right) water level statistics.

significant flood events, water levels were reduced on some reservoirs prior to the incoming flood, which helped keep the flood flows within the full bank capacity of the downstream channel.

Alternative stochastic inflow series

The principal shortcoming of the statistics provided in Figs 10–12 is that they are based on only 35 years of historical data. It should also be noted that the first 2 years in the 35-year series are among the driest on the record, which produced

unusually low water levels using the starting water levels in 1981, that were significantly below average for most reservoirs. To remove the bias and limitations associated with this particular series, a stochastic series of runoff estimates and precipitations was generated for a hypothetical 1000-year time series, using a previously published stochastic generator (Ilich 2013). All relevant statistics (means, standard deviations, probability distributions, significant lag cross-correlations and auto-correlations) for both the selected time steps lengths and on an annual basis have been preserved. For brevity, only annual

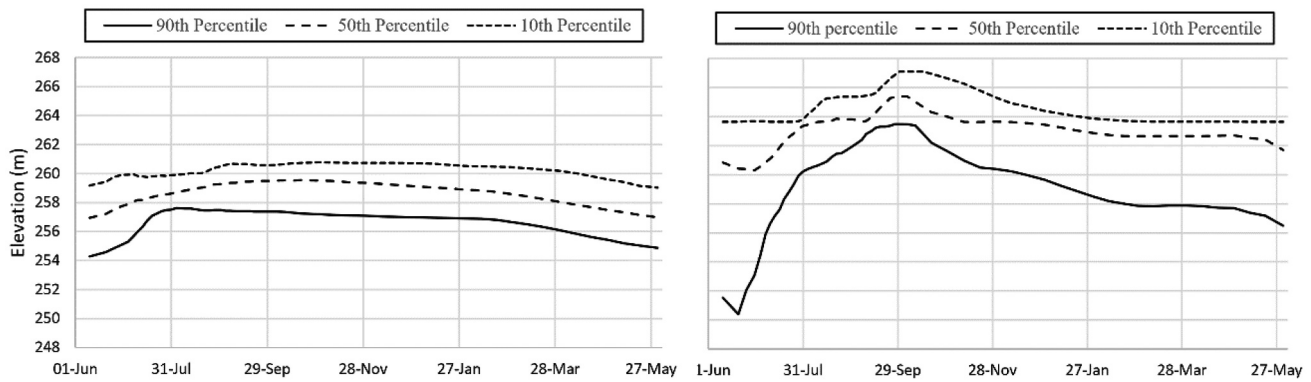


Figure 12. Tenughat reservoir historical (left) and simulated (right) water level statistics.

statistics are shown in Table 1, which are matched reasonably well although the stochastic model does not use the decomposition principle.

Based on the 35 years of historical data that were used as input into a stochastic model, a 1000-year stochastic series of flow and rainfall data were generated in this study and used as input into the optimization model to obtain 1000 years of perfect model responses, i.e. to produce the best possible reservoir operation with assumed perfect forecast over each of the statistically possible 1000 hydrological years. These model solutions can then serve as a learning database for analysing reservoir operating rules. Although statistically similar to the

historical series in terms of probability distribution functions and other relevant statistics, the stochastic series has drier and wetter hydrological years compared to the relatively short 35 years of historical record. It also has more occurrences of back-to-back dry or wet years, thus posing a greater challenge for the optimization solver to find the best solution in each simulated year. Only one out of 1000 years resulted in flooding the valley below Durgapur Barrage. Comparisons of statistical storage levels obtained from the 35 years of historical flows (which were compared to historical operation in Figs 10–12) and statistical analyses of the 1000 years of optimal solutions based on stochastic flows are shown in Figs 13–16.

Table 1. Annual cross-correlations between all reservoir inflows and precipitations.

	Historical local runoff into major reservoirs					Historical precipitation on major reservoirs				
	Panchet	Maithon	Konar	Tenughat	Tilaya	Panchet	Maithon	Konar	Tenughat	Tilaiya
Panchet	1	0.823	0.812	0.917	0.688	0.472	0.725	0.786	0.705	0.658
Maithon		1.000	0.886	0.765	0.757	0.325	0.766	0.741	0.544	0.564
Konar			1.000	0.748	0.800	0.326	0.727	0.862	0.554	0.544
Tenughat				1.000	0.603	0.519	0.732	0.793	0.752	0.723
Tilaya					1.000	0.284	0.698	0.746	0.497	0.648
Panchet						1.000	0.571	0.481	0.598	0.552
Maithon							1.000	0.832	0.741	0.735
Konar								1.000	0.739	0.767
Tenughat									1.000	0.788
Tilaiya										1.000
	Stochastic local runoff into major reservoirs					Stochastic precipitation on major reservoirs				
	Panchet	Maithon	Konar	Tenughat	Tilaya	Panchet	Maithon	Konar	Tenughat	Tilaiya
Panchet	1.000	0.813	0.806	0.884	0.760	0.622	0.703	0.792	0.669	0.660
Maithon		1.000	0.846	0.744	0.754	0.558	0.731	0.727	0.592	0.586
Konar			1.000	0.738	0.740	0.615	0.713	0.831	0.630	0.572
Tenughat				1.000	0.744	0.666	0.701	0.752	0.736	0.685
Tilaya					1.000	0.582	0.709	0.703	0.623	0.658
Panchet						1.000	0.757	0.727	0.774	0.715
Maithon							1.000	0.761	0.782	0.714
Konar								1.000	0.747	0.712
Tenughat									1.000	0.750
Tilaiya										1.000
Mean annual flows and precipitation										
Historical	195.08	178.11	26.88	126.07	18.28	20.26	20.12	18.98	19.27	19.47
Stochastic	195.96	180.10	27.18	127.77	18.72	20.44	20.23	19.14	19.42	19.68
Standard deviation of annual flows and precipitation										
Historical	60.27	50.09	7.65	52.79	10.37	3.05	4.62	5.46	3.24	4.79
Stochastic	74.27	61.73	9.64	69.01	12.33	7.80	7.51	8.01	7.05	8.00

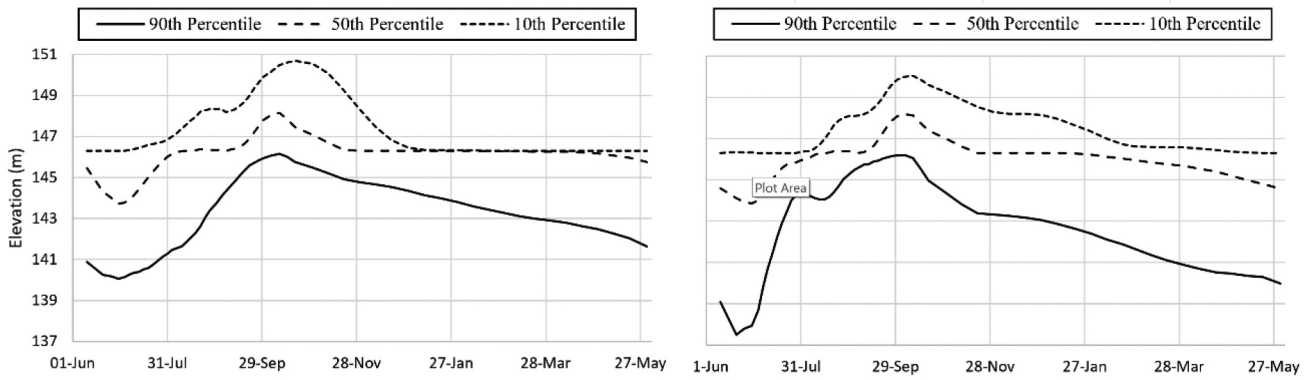


Figure 13. Maithon reservoir stochastic (left) and historical (right) optimal solutions.

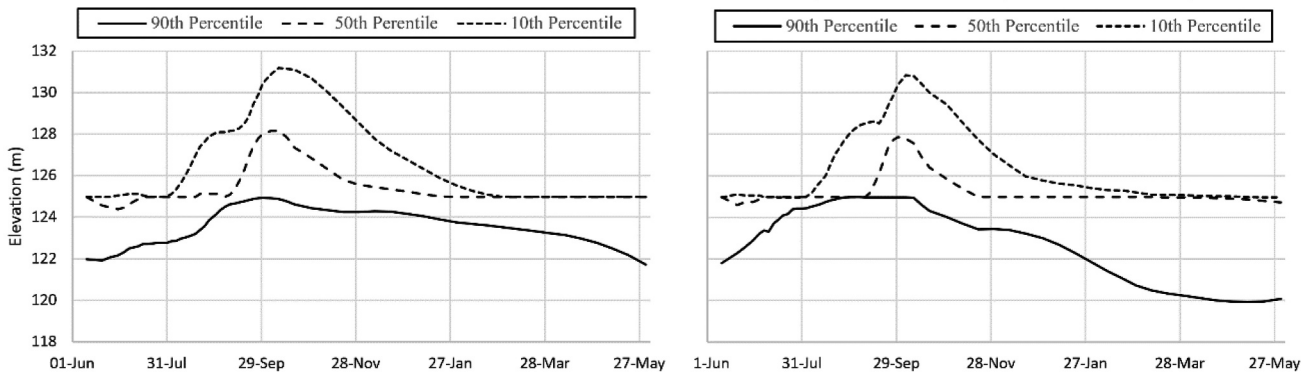


Figure 14. Panchet reservoir stochastic (left) and historical (right) optimal solutions.

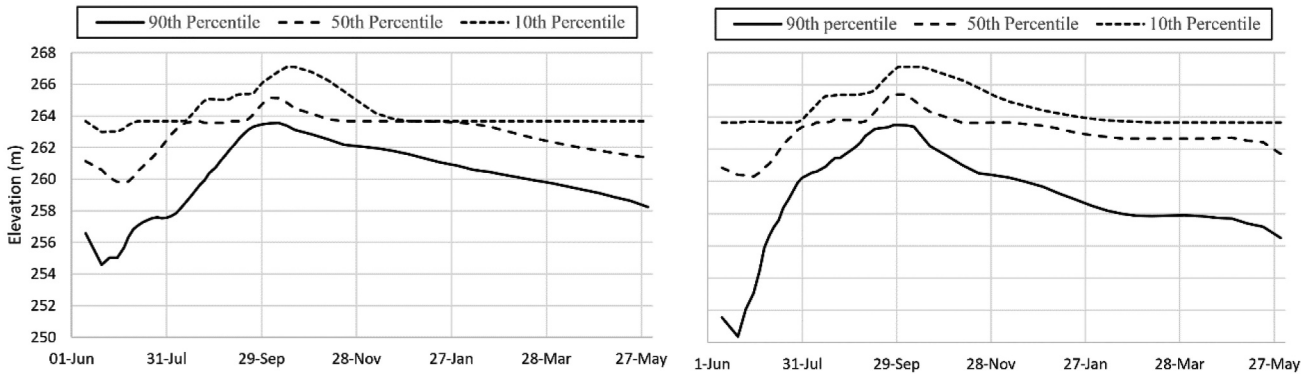


Figure 15. Tenughat reservoir stochastic (left) and historical (right) optimal solutions.

The result of removing the bias of a short historical record is seen in the stochastic scenario, where reservoir levels are maintained a bit higher than in the historical scenario, and where the statistical percentile curves tend to be smoother. This bias is due to the statistically small sample of only 35 years, compounded by a combination of low starting storage level and very dry hydrological conditions in the two starting years. This bias can be corrected by using a much longer statistical sample of solutions based on the stochastic input series.

Conceptual development of an optimization model with short-term inflow forecasts

A short-term operational model with hydrological forecast available over two simulated time steps was based on the use of reservoir operating zones shown in Figs 17 and 18. The normal water level (NWL) for the zones corresponds to the 50th percentile elevations for the end of each time interval, obtained from the stochastic optimizations with 1000 years of data that are shown in Figs 13–16. There are two zones above the NWL and five zones below it, with the

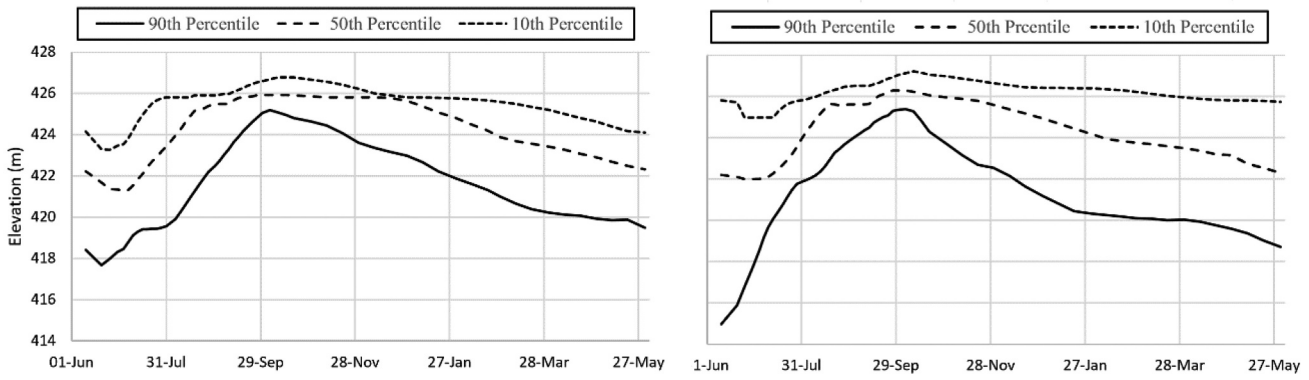


Figure 16. Konar reservoir stochastic (left) and historical (right) optimal solutions.

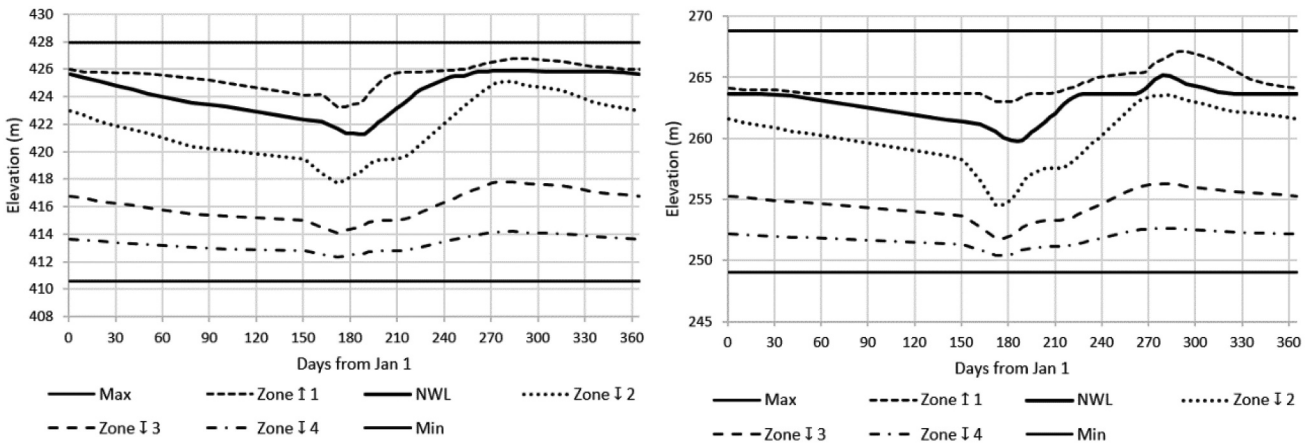


Figure 17. Operating zones for Konar (left) and Tenughat (right) reservoirs.

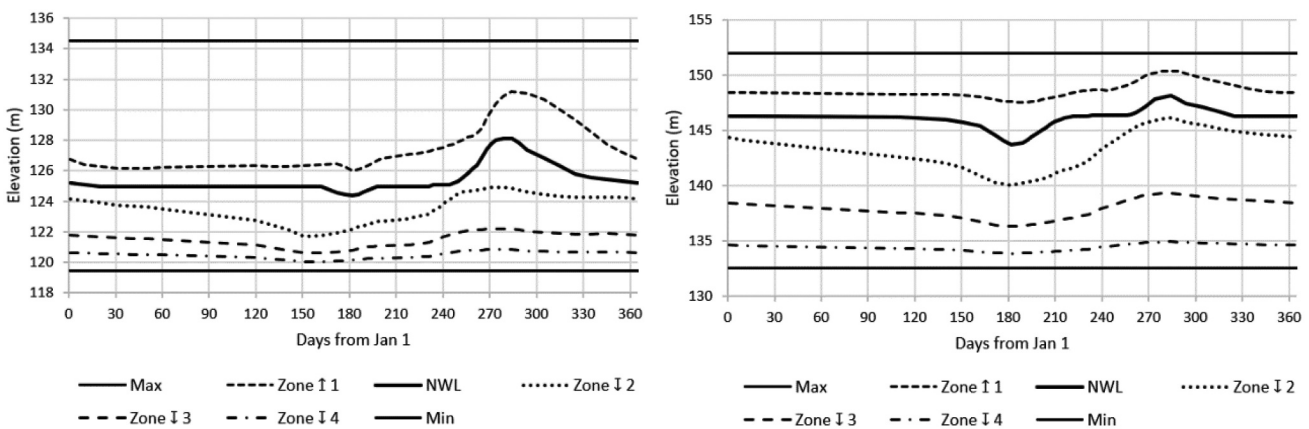


Figure 18. Operating zones for Panchet (left) and Maithon (right) reservoirs.

fifth zone below NWL representing the dead storage zone, designated as the minimum operating level, while the top zone above NWL is designated as the maximum permissible water level. The following reservoir operating rules are in effect while executing MTO solutions for two consecutive time steps:

(1) Under sufficient water supply conditions, reservoirs should remain at their NWL.

- (2) If reservoir inflow exceeds the capacity of the hydro-power plant while the storage is at NWL, the model will allow excess inflow to be stored temporarily up to the top of the first zone above NWL while discharging outflow through the turbines at their capacities.
- (3) Both the first zone above NWL and the top flood control storage zone can be used during floods when reservoir outflow needs to be kept within the full bank capacity of the downstream river reach below

Durgapur. To achieve this, the model may draw the storage down in the first two time steps, which are solved simultaneously to any elevation between NWL and the minimum water level, and refill the storage in the second time interval up to the maximum water level in an effort to keep the downstream flood flows within the full bank capacity.

- (4) When releases are made for downstream demands, Konar and Tenughat reservoirs will be drawn first, to meet their respective municipal and irrigation demands located upstream of the Panchet Dam. Following that, additional releases may be made to maintain storage levels at Panchet so as to assist with maximizing generated hydropower.
- (5) During dry seasons, water levels in all reservoirs will be kept within the zone of the same order (i.e. all reservoirs will be at the bottom of zone 1 before one of them dips into zone 2, and the same rule applies for transition from zone 2 to 3, and 3 to 4), with the drawdown starting at Konar and Tenughat first, followed by Maithon and Panchet. Only when all reservoirs are at the bottom of zone 2 can withdrawal from zone 3 begin, first at Konar and Tenughat, followed by Panchet and Maithon. Tilaiya reservoir follows the same rules, except that its zones are found to be very close to the NWL, due to low inflows, which necessitates maintenance of high water levels. The best way to utilize Tilaiya reservoir was to insure it provide supplies to the local domestic and industrial needs, and maintains storage levels as close as possible to the NWL.
- (6) If all reservoirs are in their second zone below NWL, irrigation supply will be cut by 10% of the target demand.
- (7) If all reservoirs are in the third zone below NWL, irrigation supply will be cut by 20% of the target demand.
- (8) If all reservoirs are in the fourth zone below NWL, irrigation supply will be cut by 30% of the target demand.
- (9) Once the reservoirs are at the minimum level and there is no inflow, irrigation supply will be cut to zero.

In real-time applications, the use of MTO solutions for two consecutive time steps would be applicable only for the first of the two time steps. The forecast for the second time step would have an effect on the model solution of the first time step, but the model solution (reservoir releases) would be applied in real time only for the first time step. At the end of the first time step, the operators would run the model again with updated

storage levels (requiring information from the field) and updated inflow forecasts over the next 6 d, which are simulated in this study as two consecutive time steps, each having a duration of 3 d. The duration of time steps is longer in the dry season, but it is also easier for flow forecast, since they are no longer based on rainfall–runoff transformation, but rather on the base flow hydrograph recession analyses. Also, real-time mode application would likely involve daily time steps if the forecasts are available on a daily basis. In that case, MTO solutions would be derived for 6 d ahead based on the forecasts, but only the solution for the first of the 6 d would be applied to reservoir operation.

If the above operating rules are followed and the reservoir releases are based on the MTO solutions for two consecutive time steps, the model can achieve very close performance to the optimum obtained by using the perfect forecast for the entire hydrological year, as shown in Table 2.

The model produces on average 63% more hydropower relative to the historical production, and allocates 350 million m^3 more water to users below Durgapur, while at the same time it manages to always keep flows downstream of Durgapur below $3300 \text{ m}^3/\text{s}$. All that is required to achieve this is the use of an optimization model and a reliable flow forecast for 6 d lead time.

Use of a pattern-matching algorithm to develop reservoir operating guidelines

There are numerous attempts in the literature to use artificial intelligence algorithms that can learn from the large database of optimal solutions and provide an informed guess regarding the target reservoir outflow, based on matching the current conditions in the field with the conditions of the solutions in the database. Having a hypothetical 1000 years of optimal solutions can enable basin managers to match the current reservoir level in the field for a particular time of the year with the database of optimal solutions, and match a smaller number of perfect solutions to the current conditions in the field in terms of the starting reservoir levels and recent reservoir inflows (typically over the past 90 d). Recent work by Gavahi *et al.* (2019) applies similar ideas by using the adaptive neuro-fuzzy system to set reservoir releases, an algorithm which is hard to explain to most reservoir operators. Regression approaches have been used in the past in an effort to predict storage releases as a function of the starting reservoir levels and inflow forecasts, but it is felt that better and more stable machine learning algorithms can be devised to aid future reservoir operations. The algorithm investigated here involved matching the starting reservoir level from the field with the

Table 2. Summary comparison of mean annual historical and simulated basin operation.

Scenario	Tilaiya (GWh)	Maithon (GWh)	Panchet (GWh)	Total water use (10^6 m^3)	Number of years ^a with floods (i.e. $Q > 3300 \text{ m}^3/\text{s}$ below Durgapur Barrage)
Historical Verification	10.49	180.62	205.29	4751	6
Optimal MTO	12.86	300.89	393.23	5222	0
6 d flow forecast	10.18	276.37	358.23	5100	0
Installed power (MW)	4	60	80	-	-

^aThe number of simulated years was 35, hence the historical flood occurrence was $6/35 = 17\%$ of the time. MTO = multiple time step optimization.

simulated reservoir levels for the same time of the year out of 1000 optimal solutions, and selecting the solutions that have similar inflows in a number of previous simulated time steps. In simple terms, this approach can be described as pattern matching of the available solutions for 1000 years of stochastic inflows with the current reservoir levels in the field and with the previous reservoir inflows that have been observed in a given period, and it proceeds according to the following steps:

- (1) Apply the filter to select a subset of solutions with reservoir levels that are sufficiently close to the starting reservoir level based on the information from the field;
- (2) Apply the second filter to the results of the previous filter to identify solutions that also have similar inflows into the reservoir over a specified previous period (typically 60–90 d). Applying both filters 1 and 2 usually brings the number of selected solutions from the initial pool of 1000 down to less than 50 that should be considered for further statistical analyses.
- (3) Analyse the solutions that have passed both filters to identify their 50th percentile elevation at the end of the current time step. Assume this solution to be the terminating solution for the time step.

- (4) With the assumed ending reservoir level determined in step 3, move to the next time step, and repeat the whole process (steps 1, 2 and 3) for the subsequent time steps in a simulated year.

The above approach was tested and compared with the optimal simulation for various historical years, as shown in Figs 17 and 18. The results are encouraging for years that have close to average hydrological conditions, but they also highlight the importance of taking into account the uniqueness of each hydrological year and the need to rely on runoff forecast as much as possible.

The graphs in Figs 19 and 20 show general agreement with water levels predicted based on the current conditions (starting storage and recent inflows) and similar solutions found in the database of 1000 years of optimal solutions. The solid line shows the best solution obtained using the MTO approach for a particular historical year, while the range of expected solutions based on the proposed algorithm should be between the 10th and 90th percentile dashed lines. While the historical optimal falls within this range on average 80% of the time, it does not capture individual variations that are caused by the combinations of runoff and demands on all storage reservoirs. Storage drops on Maithon in 2012 faster than predicted from

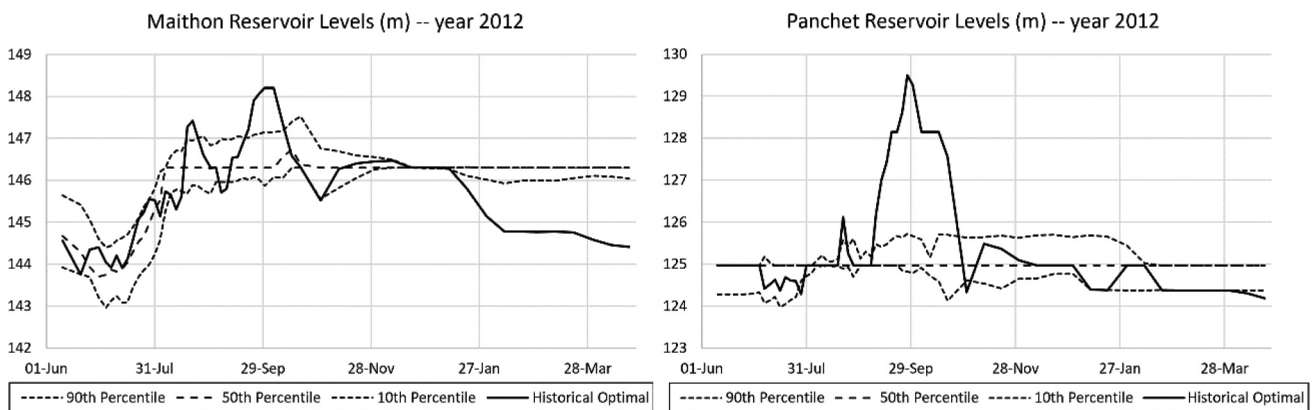


Figure 19. Comparison of predicted and solved reservoir levels for Maithon and Panchet in 2012.

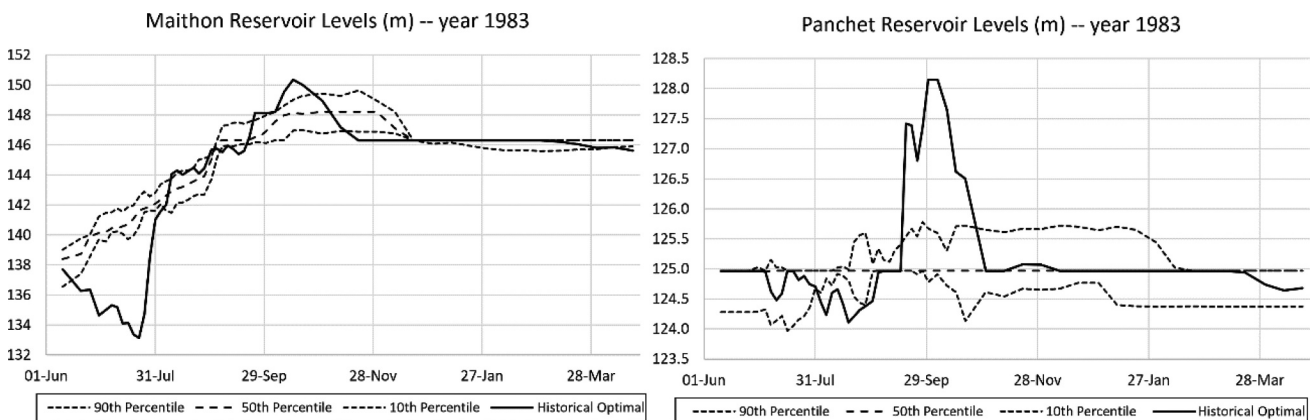


Figure 20. Comparison of predicted and solved reservoir levels for Maithon and Panchet in 1983.

February to April, and similarly in June/July of 1983, due to delayed onset of the monsoon season. Similarly, the model fails to properly predict the levels of Panchet reservoir, compared to the optimal historical simulation. Failure to predict optimal water levels with reasonable accuracy all the time using statistical inference should not come as a surprise in a complex system with multiple reservoirs and varying runoff and water demand conditions throughout the basin. Damodar basin has a large year-to-year variation in the historical hydrographs and it may have multiple peaks within a hydrological year. This reinforces the need to use short-term flow forecasting in combination with an optimization model that is built on easy-to-follow operating rules. Better use of artificial intelligence can improve rainfall and runoff forecasts.

Discussion

The proposed MTO method delivers superior results compared to the inferential pattern-matching method. However, the data requirements for the MTO method are more demanding. In addition to the starting reservoir storage, the MTO method requires a reliable short-term runoff forecast, and an optimal storage sharing policy among reservoirs, which can only be developed from the previous planning study based on the application of MTO over long-term hydrological inputs. Long-term historical optimization in this project required a model that can optimize the entire river basin with variable time step lengths, non-linear constraints related to the maximum flows through the turbines and net evaporation on reservoirs, and a large-scale solver capable of delivering reliable solutions within a reasonable computational time. There are 35 years of historical data available, with each year having 60 time steps and 105 variables in each time step, which amounts to an optimization problem with 220 500 variables. There is no alternative to LP when it comes to finding the best possible solution to this problem, especially since LP guarantees finding the global optimal solution, and it does so within 10 minutes. Furthermore, there is no heuristic solver that can even begin to solve an optimization model of this size while maintaining all constraints within the feasible range and guaranteeing optimality. Only four LP-based models are capable of calculating MTO solutions: HEC-ResPRM (Hydrologic Engineering Centre 2020), which is free but only works with a monthly time step; RiverWare (Zagona *et al.* 2001), which requires the model to run time steps of equal length, thus making it ineligible for this application, in addition to its prohibitive cost; OASIS (Randall *et al.* 1997), which is proprietary and very expensive; and WEB.BM (Ilich 2019) which is available for free. These considerations made the choice of the latter model obvious.

Potential application of this approach in real-time reservoir management is subject to the accuracy of the runoff forecasts, which are always uncertain. Another limitation is that the current study uses 3-d time steps. An operational model should be tested using daily time steps and real-time forecasts, and the daily time steps could reduce the length of the required forecast period to 5 d subject to additional testing. However, daily time steps require hydrological routing, and the required input data for hydrological routing were not available at the time this study was conducted. The National Hydrology Project office has initiated negotiations between the Damodar

Valley Corporation and the Central Water Commission of India to expand this study and link the WEB.BM model to the rainfall–runoff forecasting model previously developed by the Danish Hydraulic Institute for this basin. The main driving force for further development are the WEB.BM model results on testing over the past 35 historical years – flood damage from six historical floods could have been eliminated, and water supply might have been increased by 350 million m³ per year on average (or by 7.4% compared to the historical average), while simultaneously producing 63% more hydropower per year compared to the historical operation. These results are based on the implementation of the zoning concept during normal operation combined with the MTO-driven reservoir releases, and a reliable runoff forecast for up to 6 d ahead combined with MTO-driven reservoir operation in real time, which is a small investment considering the potential benefits.

Conclusions and recommendations

River basin modelling has so far typically been based on the use of user-supplied reservoir rule curves, which were not developed in a scientific way. The use of optimization models in combination with stochastic models can be very effective in constructing reservoir operating rules in a scientific way. This strategy can be combined with short-term runoff forecast and with short-term optimization to guide future reservoir releases in real time. Possible improvements demonstrated in this study point to the potential for significant flood damage reduction along with increased generation of hydropower. The recommendations resulting from this study can be summarized as follows:

- (1) Optimization models should first be used to help develop river basin plans, before attempting to apply them as seasonal operational tools. A good plan is based on comprehensive efforts to develop historical time series of the available runoff and provide all other background hydrological and water demand analyses, in addition to analysing the optimization model results.
- (2) The success of applying optimization tools in real-time operation will depend on the improved ability to forecast incoming runoff and precipitation over short time horizons, and artificial intelligence and machine learning algorithms should be focused on the issue of runoff forecasts, rather than on the issue of trying to predict the best reservoir management decisions.
- (3) Based on the results of this study, the tested pattern-matching algorithm may be useful in providing approximate guidelines for operators in terms of the typical anticipated range of reservoir levels for specific times of the year, but it cannot address individual hydrological events, which can have significant spatial and temporal variation.
- (4) Significant improvements to future water management are possible with better planning studies based on the use of optimization models with MTO capabilities, better forecasting systems, and the use of MTO optimization models in real time to help implement management plans and manage the forecasted inflows in the best possible way.

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Disclosure statement

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