Limitations of Network Flow Algorithms in River Basin Modeling

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Abstract: A number of computer models for river basin planning and management have been developed by various agencies and used extensively since the mid-1970s. Most of the early developments have been based on the use of heuristic weight factors to represent priorities of allocation, and specialized optimization algorithms that were based on the use of network flow algorithms (NFAs). While these algorithms were at first considerably faster than the standard Simplex solvers, their handling of flow constraints was simplistic, which eventually led to the use of iterative schemes for handling nonnetwork constraints. This paper critically examines the notion that iterations applied in combination with NFA are a good vehicle for handling nonnetwork constraints. The failures are demonstrated on several variants of a simple problem with two reservoirs in series.

DOI: 10.1061/(ASCE)0733-9496(2009)135:1(48)

CE Database subject headings: River systems; River basins; Resource management; Reservoir operation; Mathematical models; Optimization models.

Introduction

Early efforts to use computer models in the water resources field attempted to capture the essence of rainfall-runoff transformation, or to model a propagation of flood waves using the known mathematical relationships that describe these processes. River basin management modeling brought in an additional degree of complexity, requiring that the modelers identify various types of water use and handle different allocation priorities and deficit sharing policies among them. Depending on the priorities, the available flow could completely bypass an upstream user and be allocated to a downstream user, or vice versa. A major departure from previous modeling of physical processes was the need to either define a complex set of rules that account for every possible combination of supply and demand conditions, or to rely on the model to find the best way to regulate flows in the system, given the priority of supply assigned to each water use. To this end, a frequent approach was to use a mathematical optimization solver to address finding the best basin-wide water allocation. A review of reservoir operation models for basin planning purposes was compiled by Wurbs (1993) and subsequently updated by Labadie (2004). This paper deals with the models based on network flow algorithms (NFAs). There are a number of such models and they have been used in many practical applications.

The priority of supply represents the water licensing system in North America from where most of the early model development originated. A water licensing system can be represented using a linear programming (LP) formulation. Hence, early efforts focused on the search for efficient LP solvers with typical objectives of finding an optimal set of network flows. One of the first such solvers that was initially used in a number of basin planning models was the out-of-kilter algorithm (Fulkerson 1961). Consider a network consisting of ordered pairs (i,j) of arcs A and a total of N nodes i,j. The minimum cost flow problem is defined as the following linear (network flow) program:

$$\text{minimize} \sum_{(i,j)\in A} c_{ij}x_{ij} \quad \forall\, i,j \in N \tag{1}$$

$$\text{subject to: } \sum_i x_{ij} - \sum_i x_{ji} = 0 \quad \forall\, i \in N \tag{2}$$

$$0 \leqslant l_{ij} \leqslant x_{ij} \leqslant u_{ij} \quad \forall\, (i,j) \in A \tag{3}$$

where c_{ij}, l_{ij}, x_{ij}, and u_{ij} are respective cost (or value) factors per unit of flow, lower bound, flow, and the upper bound on flow along an arc (i,j). Constraint (2) represents the mass balance at each node. Expression (1) is a general definition of the minimum cost flow problem in the operational research literature. In water resources allocation, the aim is to maximize supply to all users according to their respective priorities. The minimization problem given by expression (1) can be converted to a maximization problem in two ways:

(a) by assigning negative sign to the cost factors c_{ij}; or,

(b) by using the goal programming formulation, where the penalties retain their original values but the objective function is modified to minimize the sum of all deficit flows (i.e., deviations from ideal targets defined as the difference between the upper bounds u_{ij} and flows x_{ij} on each arc). Formulations (a) and (b) are equivalent, since

$$\min \sum_{(i,j)\in A} c_{ij}(u_{ij} - x_{ij}) \Leftrightarrow \max \sum_{(i,j)\in A} c_{ij}x_{ij} \quad \forall\, i,j \in N \tag{4}$$

To apply formulation (b) to water resources networks while retaining the objective function form defined in (1), it is necessary to introduce an additional ideal arc (i,j) for each water use component. Such an arc has its upper and lower bounds u_{ij} and l_{ij} set

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Note. Discussion open until June 1, 2009. Separate discussions must be submitted for individual papers. The manuscript for this paper was submitted for review and possible publication on May 29, 2007; approved on May 20, 2008. This paper is part of the ***Journal of Water Resources Planning and Management***, Vol. 135, No. 1, January 1, 2009. ©ASCE, ISSN 0733-9496/2009/1-48–55/$25.00.

to the ideal target value for a given time step and its cost set to zero. One (or more) arcs with reversed direction (j,i) are used to enable the reduction of the ideal flow during limited supply conditions, and such reductions are penalized per unit of flow x_{ji}. Due to the reversed direction of this arc with respect to the ideal arc, their combined effect on network solution is equivalent to the deficit flow u_{ij}-x_{ij}. Yet both the ideal and the reversed arcs maintain nonnegative flows and costs. Hence, in such representation, the objective function retains its original form (1), while the network configuration ensures the goal programming summation defined in (4). A perfect flow solution would then have a total cost of zero, with flows in all components being at ideal levels while all reversed arcs contained zero flows, implying no deviations from targets.

Although the values of the objective functions are different for formulations (a) and (b) above, the actual network flow solutions are the same for identical problem definition. More information is available in publications by Sigvaldason (1976), which described the acres reservoir simulation program (ARSP). It should also be noted that some network flow algorithms provide an option to solve the maximization problem directly, without having to resort to the change of sign of the cost factors c_{ij} or to the introduction of additional ideal arcs as in the case of the goal programming formulation.

Early river basin planning models that utilized NFA were SIMYLD (Evanson and Moseley 1970), ARSP (Sigvaldason 1976), MODSIM3 (Labadie et al. 1986), WASP (Kuczera and Diment 1988), DWRSIM (Chung et al. 1989), CRAM (Brendecke 1989), KCOM (Andrews et al. 1993), and WRMM (Ilich et al. 2000). Most of these models are still in use, and some early versions have evolved to more sophisticated models such as CALSIM (Draper et al. 2004), which uses a mixed integer programming (MIP) solver, and it is still maintained and used actively by the California Department of Water Resources. In some models, such as the SIMYLD or the WRMM, the original version of the out-of-kilter algorithm has been replaced with alternative variants, which were proven to be significantly faster such as the SUPERK algorithm of Barr et al. (1974) or, as in the case of the MODSIM model, the Relax4 network flow solver of Bertsekas and Tseng (1988), which is also used in the REALM model (Perera and James 2003; Perera et al. 2005). Another model developed in Australia in addition to REALM is the WATHNET (Kuczera 1992), which also uses a specialized network flow solver known as the simplex-on-a-graph algorithm, developed by Kennington and Helgason (1980).

The NFA-based models have been used extensively in river basin planning studies. Nonnetwork constraints have been handled by using successive iterations within a time step until a desired convergence is achieved. This is done by initially guessing the flow bounds u_{ij} on dependent components, solving the minimum cost flow problem, evaluating the network flow solution against the assumed bounds, resetting the bounds to new values based on the previous solution, and reiterating if necessary until the assumed bounds and the network flow solution were within a reasonable tolerance limit (Ilich 1993). Although the use of iterations is openly acknowledged in the accompanying documentation of some of the models such as MODSIM, ARSP, or REALM, no information is available on its success, the rate of convergence, or the impact of iterations to the quality of the converged solutions. The iterative process is typically associated with resetting the reservoir outflow limits, diversion flows at unregulated weirs where maximum flow diversion is a function of the available flow in the river, irrigation return flows, or hydropower output or canals losses. In other words, any instance where the flow in one component is related to the flow of another component. Such conditions are generally described as nonnetwork constraints, since they do not comply with the problem formulation given by Eqs. (1)–(3). Nonnetwork constraints can be linear or nonlinear. Examples of linear nonnetwork constraints are canal losses expressed as a linear function of canal flow, or irrigation return flows expressed as a linear function of consumptive use. Examples of nonlinear nonnetwork constraints are reservoir outflow limits as a dynamic function of storage, or diversion canal flow limits expressed as a function of the available river flow that determines the water level at the head gate. These constraints can be linearized, which leads to some loss of accuracy in the piecewise linear segmented representation of nonlinear functions. Although examples in this paper rely on the use of nonlinear constraints, the principal cause of failure presented here is not associated with the loss of accuracy due to linearization, but rather with the inability to include nonnetwork constraints directly into the search process. This view can be confirmed if one assumes that the reservoir outflow is a linear function of storage, and that the shape of the storage reservoir is cylindrical, resulting in constant rate of head change for the average net outflow over a time step. Even with these assumptions, the use of NFA with iterations on sample problems presented in this paper would still fail to deliver the best possible solutions by a wide margin.

When iterations are used, the resulting solution from one iterative call of the solver becomes the starting point for the next iteration. This paper demonstrates that there is no guarantee that this process results in a convergence to the global optimum even on simple systems with two or more sequential reservoirs.

Attempts to include nonnetwork linear constraints directly into the LP formulation have been made using specialized solvers such as the EMNET (Sun et al. 1995) or using general LP solvers as in the case of the RIVERWARE (Zagona et al. 2001), HEC-FCLP (Needham et al. 2000), or OASIS (Dean et al. 1998). These models are capable of optimizing allocation over single or multiple time steps. There seems to be no universally accepted methodology on how to utilize multiple time step solutions for the development of practical short-term operating rules, since multiple time step solutions require the perfect forecast of hydrologic inputs over long periods that are unavailable in real time. Simultaneous multiple time step solutions can also be obtained with HEC-PRM (Lund 1996), but the nonlinear constraints can only be handled using iterative schemes, as this model still relies on an NFA algorithm.

Typical modeling time steps used for planning purposes are weekly or monthly, since the time step length has to be sufficiently longer than the time of travel through the entire region of the river basin under consideration. Test problems in this paper are presented for a single weekly time step. Some of the tests demonstrate that shortening the time step does not resolve the issues raised in the paper.

Early versions of network flow models dating back to the 1970s did not originally include iterative mechanisms for modeling nonnetwork constraints. This is no longer the case, as most NFA-based models in use nowadays employ some form of iterative scheme, for example to account for the hydraulic dependence between the maximum reservoir outflow and the available storage, or to handle other dynamic constraints where flows in one component are dependant on flows in other components in the network. One prominent example of nonnetwork constraints are reservoir outflow limits, which are governed by the level of stor-

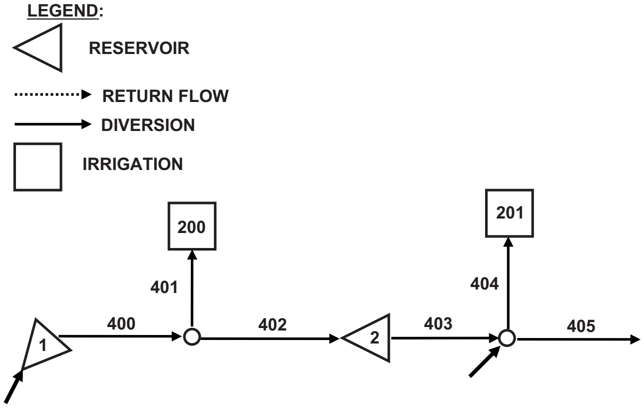

Fig. 1. Problem 1 modeling schematic

age (i.e., another variable in the model), and the capacity of the outlet structure (either a weir or an orifice). Reservoir elevation determines the maximum possible outflow at each point during the given time period. Yet the elevation also changes during the time period as a result of the overall mass balance of inflows and outflows. Further, many reservoirs have more than one outlet structure, which can compound the problem when they are operated simultaneously. The average outflow capacity over a time step is the integrated average from the beginning to the end of the calculation time step. For example, if $Q(V)$ represents the maximum reservoir outflow Q as a function of storage V at any given moment (defined by a weir or an orifice equation), then the maximum possible outflow $Q_{\max}$ over a time step t can be expressed as

$$Q_{\max} = \frac{1}{T_e - T_i} \int_{T_i}^{T_e} Q[V(t)]dt \qquad (5)$$

where T_i and T_e=starting and ending times for a given time interval. Only $Q(V)$ at the initial time T_i is known, since the storage may be reduced or increased in a given time step, which is to be determined as part of finding the overall solution. When using a network flow solver, the model must "guess" the final elevation for a time step in order to numerically integrate the average outflow capacity given by Eq. (5) and, thus, estimate the upper bound on reservoir outflow. Hence, the need to resort to iterative calls of the network flow solver, such that the guessed value is improved from iteration to iteration until it is sufficiently close to the integrated value. Once $Q_{\max}$ in Eq. (5) is set, it defines the upper bound on the outflow for a single iteration. The model may actually derive outflows that are less than $Q_{\max}$ since reservoir releases are driven by downstream demands, which are usually less than the maximum possible release ($Q_{\max}$) defined by Eq. (5).

In LP formulation, all water movements are expressed in the same units, which are typically either units of flow or volume. Which of those units are used is irrelevant, since the physical nature of the problem persists in the same way. An empty reservoir cannot provide desired gravitational water releases until some storage is first refilled to provide sufficient head for such releases. The storage versus outflow curve can be given with both storage and outflow in the units of volume or in the units of flow for a given time step length. Some models such as REALM allocate water using the units of volume also rely on iterative convergence schemes in the solution process.

Table 1. Storage and Outflow Capacities for Reservoirs 1 and 2

Volume ($1{,}000\ m^3$)	Elevation (m)	Outflow (m^3/s)
0.00	1,653.00	0
772.03	1,656.00	0
1,960.51	1,659.00	0
2,400.00	1,660.00	0
2,900.00	1,661.03	1.8742
3,400.00	1,662.01	3.2872
3,900.00	1,662.92	4.2828
4,400.00	1,663.76	4.9314
4,900.00	1,664.52	5.2931
5,400.00	1,665.20	5.4391

Table 2. Inflows for Reservoirs 1 and 2

Location	Inflow from runoff (m^3/s)
Reservoir 1	6.0
Reservoir 2	1.5

Test Problems

The schematic for test problems is shown in Fig. 1. It contains two reservoirs, numbered 1 and 2, and two water use components (e.g., irrigation) numbered 200 and 201. Channels 400 through 405 define network configuration. Several test runs are conducted on the schematic in Fig. 1 and each test run is only conducted for a single time step with a duration of one week. Input data for this test run are given in Tables 1–5 including the storage and outflow capacities (Table 1), inflows (Table 2), initial and ideal storage levels (Table 3), water demands (Table 4), and priorities of allocation (Table 5). Live storage on both reservoirs begins at the elevation of 1,660 m. In this test no restrictions on reservoir outflows are assumed, and a common sense policy of sequential reservoir operation (Lund and Guzman 1999) is adopted with the downstream reservoir having a lower priority than its upstream counterpart. Hence, storage in reservoir 2 has the lowest priority, followed by reservoir 1, water use (irrigation) component 200, irrigation component 201, and finally channel 405 with the highest priority. Low priority on storage helps define the value of storage. Without it, the model would have no incentive to refill storage when surplus runoff is available. In this test run, the values representing those priorities are 1, 2, 10, 100, and 200 for reservoirs 2, 1, irrigation blocks 200 and 201, and channel 405, respectively. Hence, the cost vector c_{ij} given in expression (1) contains values (1, 2, 10, 100, and 200) for components 2, 1, 200, 201, and 405, respectively. If the optimization problem is solved as minimization, the sign of the values of the cost vector will be reversed to negative, and the solution values x_{ij} would be identical as if the same problem was solved as maximization, as is well established in mathematical programming theory. For purposes of

Table 3. Initial and Full Supply Level for Reservoirs 1 and 2

	Initial (m)	Full (m)
Reservoir 1	1,661.0	1,665.2
Reservoir 2	1,660.5	1,665.2

Table 4. Water Demands for Test Problem 1

Component	Water demands (m^3/s)
Block 200	3.0
Block 201	3.0
Channel 405	1.0

further discussion, assume the objective function expressed in maximization form, such that the objective function for all test problems presented here can be written as

$$\max\{1 \cdot X_2 + 2 \cdot X_1 + 10 \cdot X_{200} + 100 \cdot X_{201} + 200 \cdot X_{405}\} \quad (6)$$

where X_1 and X_2 represent storage levels at the end of the week; X_{200} and X_{201} represent water supply to irrigation blocks achieved over the simulated week; and X_{405} represents average flow in channel 405 for a week. The highest value factor in the system is given to channel 405, which refers to the biological minimum target flow of 1 m^3/s, as can be seen from the water demands section of Table 4. Hence, the upper bound on the flow in channel 405 that is subjected to value factor is 1 m^3/s. The same goes for irrigation blocks 200 and 201, where water requirements are capped at 3 m^3/s of average weekly flow for each block. Therefore, the last three terms in expression (6) related to irrigation blocks and the in-stream flow needs provide the most significant input into the final value of the maximization function. If both reservoirs are empty at the end of a given time interval while all water demands are supplied in full, the objective function value is 530 ($=10 \cdot 3+100 \cdot 3+200 \cdot 1$). The ending storage expressed by the first two terms adds little to the final value of the objective function, but the difference in values between the two reservoirs helps define which of the two reservoirs should release storage first when releases for block 201 or channel 405 are required.

Test Problem 1

Table 6 provides the NFA model solution for Test Problem 1, which assumes no restrictions on reservoir outflow. In this case, the model fully meets all demands. Any balance of available inflow is kept in reservoir 1, since it has a higher priority compared to reservoir 2. Reservoir 2 is emptied, i.e., its elevation is lowered to the bottom of live storage at 1,660 m at the end of the simulated weekly time step. This solution is correct and it can be obtained either by an NFA-based model or a full LP implementation. To calculate the value of the objective function in maximization form for this test run, convert the ending storage volumes to the units of flow for a time interval to calculate the first two terms in expression (6). For example, for reservoir 2, the ending elevation is 1,660 m, which corresponds to storage of 2.4 million m^3. For the NFA model formulation, all variables (including storage) are converted to the same units, which are either average flow (m^3/s) or volume (m^3) per time step. In these examples, the model solved using the units of flow, hence, the ending storage for reservoir 2 is 3.968 m^3/s, which can be verified by dividing 2.4 million m^3 by the length of one week expressed in seconds. The storage of reservoir 1 at the end of time interval is obtained in the same manner to give 5.669 m^3/s. The final value of the objective function is 545.306 ($=2 \cdot 5.669+1 \cdot 3.968+530$), calculated using expression (6). As the title of Table 6 implies, this solution assumes no restrictions on outflow capacity as a function of storage, and as such it can be considered as the upper bound on solutions where restrictions on storage outflow are imposed.

Table 5. Allocation Priorities for Test Problem 1

Component	Rank priority	Value factor
Reservoir 1	5	1
Reservoir 2	4	2
Demand 200	3	10
Demand 201	2	100
Channel 405	1	200

Table 6. Solution to Test Problem 1—Unlimited Outflow Capacity

Component type	Component number	NFA solution	Units
Reservoir	1	1,662.066	m
Reservoir	2	1,660.000	m
Water use	200	3.000	m^3/s
Water use	201	3.000	m^3/s
Channel	400	5.099	m^3/s
Channel	401	3.000	m^3/s
Channel	402	2.099	m^3/s
Channel	403	2.500	m^3/s
Channel	404	3.000	m^3/s
Channel	405	1.000	m^3/s
Objective function		545.306	

Test Problem 2

Test Problem 2 is identical to Test Problem 1 in all aspects except that flow restrictions are imposed on channels 400 and 403 according to the outflow versus elevation relationship given in Table 1. The same curve is used for both channels 400 and 403 to simplify the setup. The aim is to include limitations on maximum outflows from storage reservoirs as a function of the available storage. The output is shown in Table 7 for all four iterations required to converge to the final solution and the value of the objective function for each intermediate solution.

The paradox about the solution in Table 7 is that, judging from the standpoint of higher priority of supply to block 201 compared to block 200, the solution given in the first iteration looks better than the converged solution in the final (fourth) iteration! This can be verified by observing that block 201, which has 10 times higher priority than block 200, is allocated 1.41 m^3/s in iteration 1 while it ends up with 0.955 m^3/s in the final iteration. In the very first iteration, the upper bound of channel 403 is set to 0.91 m^3/s based on the assumption that reservoir 2 will have no significant change of its initial elevation of 1,660.5 over the week. This assumption is evaluated after obtaining the solution from iteration 1. At the end of iteration 1, the model evaluates the maximum outflow capacity for reservoir 2 from its elevation at the beginning of the week (1,660.5 m) to its week ending elevation obtained from the solution in iteration 1 (1,660.0 m), using numerical integration of expression (5) over small time increments resulting in integrated outflow capacity of 0.455 m^3/s. The outflow capacity estimate for iteration 2 is then fixed to 0.682 m^3/s, half-way between the initial 0.91 m^3/s and the inte-

Table 7. Solution to Test Problem 2—Limited Outflow Capacity, Normal Costs

Component type	Component number	NFA iteration 1	NFA iteration 2	NFA iteration 3	NFA iteration 4	Full lp solution	Units
Reservoir	1	1,665.226	1,663.957	1,664.166	1,664.166	1,663.850	m
Reservoir	2	1,660.000	1,660.000	1,660.000	1,660.000	1,662.040	m
Water use	200	1.311	3.000	3.000	3.000	0.000	m^3/s
Water use	201	1.410	1.182	0.955	0.955	2.616	m^3/s
Channel	400	1.820	3.281	3.054	3.054	3.397	m^3/s
Channel	401	1.311	3.000	3.000	3.000	0.000	m^3/s
Channel	402	0.508	0.281	0.054	0.054	3.397	m^3/s
Channel	403	0.910	0.682	0.455	0.455	2.116	m^3/s
Channel	404	1.410	1.182	0.955	0.955	2.616	m^3/s
Channel	405	1.000	1.000	1.000	1.000	1.000	m^3/s
Objective function		375.998	367.148	344.902	344.902	481.750	

grated capacity of 0.455 m^3/s. With a reduced capacity for supplying block 201 in iteration 2, more water is available for block 200, which has a higher priority than storage in reservoir 2. Hence, in iteration 2 and all subsequent iterations, block 200 gets its full demand of 3 m^3/s, while block 201, which has the highest priority, settles for 0.955 m^3/s, less than a third of its full demand. The problem is that the NFA cannot "see" the need to keep storage at reservoir 2 sufficiently high to provide desired supply to block 201. All that the NFA can take into account are the fixed upper bounds on channel 403 from iteration to iteration and the priority factors, which drive the allocation in each iterative solution. In addition to the results of all four iterations, Table 7 also contains a column with the correct solution obtained with the full LP application. This solution better meets the objectives than the converged NFA solution shown in the column marked "Iteration 4," as it fully bypasses irrigation block 200 and maximizes water supply to block 201. To obtain the accurate solution, linearized relationships between storage and outflow must be included as additional constraints. Since these constraints imply that the flow bound is a function of other variables (i.e., storage) elsewhere in the network, they cannot be included in the network flow problem constraints formulation given by expressions (2) and (3). Objective function values are given in the bottom row of Table 7.

Test Problem 3

Can shorter time steps improve the solution in Test Problem 2? Table 8 shows converged solutions of Test Problem 3, which consists of seven sequential daily time steps instead of a single weekly time step. All other input data are the same as for Test Problem 2, with the exception of the time step length. The ending reservoir elevation for each daily solution was used as a starting elevation for the subsequent day. Hence, the ending elevation at the seventh day is equivalent to the ending weekly elevation for a weekly time step. It can be noticed that, except for the first day, the model allocates 3 m^3/s to block 200 in all six subsequent days, hence, the same process takes place as in the weekly solution, with the same consequences. Outflow of 3 m^3/s to block 200 cannot be achieved in the first day since reservoir 1 does not fill sufficiently to achieve the necessary head that could enable average daily outflow of 3 m^3/s. However, when flows for all seven days are averaged, the final solution is actually worse than the one obtained in the weekly time step. To confirm this, note that the highest priority component (block 201) gets 0.955 m^3/s in the weekly time step solution, while the average of the seven daily time step solutions results in 0.860 m^3/s. Consequently, refining the length of the time step has no positive impacts on the quality of the final solution.

Test Problem 4

An attempt to rectify the situation described in Test Problems 2 and 3 may be to reverse the values on reservoirs. For example, if reservoir 2 is assigned a higher value factor than reservoir 1, then reservoir 1 will be emptied first, leaving more water in storage for reservoir 2, thus, automatically increasing its outflow

Table 8. Daily Time Step Solutions to Test Problem 3

Component type	Component number	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Weekly equivalent	Units
Reservoir	1	1,661.631	1,662.129	1,662.601	1,663.061	1,663.497	1,663.916	1,664.310	1,664.310	m
Reservoir	2	1,660.361	1,660.260	1,660.188	1,660.135	1,660.098	1,660.070	1,660.051	1,660.051	m
Water use	200	2.285	3.000	3.000	3.000	3.000	3.000	3.000	2.898	m^3/s
Water use	201	1.283	1.065	0.907	0.794	0.712	0.653	0.609	0.860	m^3/s
Channel	400	2.285	3.000	3.000	3.000	3.000	3.000	3.000	2.898	m^3/s
Channel	401	2.285	3.000	3.000	3.000	3.000	3.000	3.000	2.898	m^3/s
Channel	402	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	m^3/s
Channel	403	0.783	0.565	0.407	0.294	0.212	0.153	0.109	0.360	m^3/s
Channel	404	1.283	1.065	0.907	0.794	0.712	0.653	0.609	0.860	m^3/s
Channel	405	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	m^3/s
Objective function		366.012	362.137	337.137	326.651	319.280	314.214	310.656	334.777	

Table 9. Solution to Test Problem 4—Limited Outflow Capacity, Reversed Reservoir Costs

Component type	Component number	Iteration 1	Iteration 2	Iteration 3	Iteration 4	Iteration 5	Iteration 6	Full lp solution	Units
Reservoir	1	1,665.226	1,663.954	1,663.677	1,663.663	1,663.663	1,663.663	1,663.850	m
Reservoir	2	1,660.000	1,660.005	1,660.641	1,660.462	1,660.307	1,660.314	1,662.040	m
Water use	200	1.311	3.000	3.000	3.000	3.000	3.000	0.000	m^3/s
Water use	201	1.410	1.182	0.964	1.121	1.246	1.240	2.616	m^3/s
Channel	400	1.820	3.285	3.577	3.591	3.591	3.591	3.397	m^3/s
Channel	401	1.311	3.000	3.000	3.000	3.000	3.000	0.000	m^3/s
Channel	402	0.508	0.285	0.577	0.591	0.591	0.591	3.397	m^3/s
Channel	403	0.910	0.682	0.464	0.621	0.746	0.740	2.116	m^3/s
Channel	404	1.410	1.182	0.964	1.121	1.246	1.240	2.616	m^3/s
Channel	405	1.000	1.000	1.000	1.000	1.000	1.000	1.000	m^3/s
Objective function		375.998	367.145	345.252	360.778	373.154	372.560	481.995	

capacity. Test Problem 4 simulation run is conducted with reversed value factors on reservoirs 1 and 2 (i.e., values of 1 for upstream and 2 for downstream reservoirs, respectively). The solutions for all iterations as well as for a full blown LP application are in Table 9.

This time the solution is slightly better, since 1.24 m^3/s is allocated to the highest priority block 201, as opposed to 0.955 m^3/s in Test Problem 2. However, not only did this change in priority of storage and release violate a sensible rule for operating reservoirs in sequence by releasing first form downstream storage, but the resulting solution is still inferior to the correct solution obtained from the full LP application that remained unchanged from the previous Test Problem 2, implying that the best possible solution is not sensitive to the reversal of reservoir values. This is because the value factor associated with water supply to block 201 is significantly higher than any of the reservoir value factors. The full LP solver can successfully maximize the objective function by allocating nothing to block 200. In the NFA solution, allocation to block 200 still persists, since block 200 has a higher value factor than reservoir 1.

Test Problem 5

Can a shorter time step improve the solution of Test Problem 4? Table 10 shows seven subsequent daily solutions for Test Problem 4, where the ending reservoir elevations for a single day are used as starting elevations for the subsequent day. As seen in Table 10, the average allocation to block 201 obtained from seven daily solutions is 1.248 m^3/s, while the allocation to block 201 for a single weekly time step in Table 9 shows 1.240 m^3/s. The difference is practically negligible. The principal cause of failure to find the optimal solution remains the same regardless of the length of the time step.

Test Problem 6

If the value factor on reservoir 2 is set to 20 and, thus, higher than block 200, which has a value of 10, the solution would closely match the one obtained by the full LP solver. The allocation to block 201 would be 2.75 m^3/s instead of 2.62 m^3/s. This is due to slightly higher accuracy of integration scheme employed in the NFA model compared to the full LP formulation, since the full LP formulation must include linearization of the integration scheme directly in the constraint matrix. However, this value setup would require that reservoir 2 be always kept full, even in cases when that is not required. That would defeat the purpose of using mathematical programming for finding optimal demand-driven allocation. Consider for example a situation in Test Problem 6 where inflow downstream of reservoir 2 is set to 4 m^3/s (or any greater value) and the value factor of reservoir 2 is set to 20. In this case, storage release of reservoir 2 is not required for block 201 and channel 405 at all, and the available storage in reservoir 1 could be used to meet the demand at block 200. However, with the second highest value factor on reservoir 2, the model forces releases from reservoir 1 to refill storage at reservoir 2, resulting in no supply to block 200 at all. The solution of the NFA model for

Table 10. Daily Time Step Solutions to Test Problem 5

Component type	Component number	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Weekly equivalent	Units
Reservoir	1	1,661.631	1,662.115	1,662.491	1,662.808	1,663.065	1,663.286	1,663.483	1,663.483	m
Reservoir	2	1,660.361	1,660.274	1,660.291	1,660.362	1,660.458	1,660.557	1,660.653	1,660.653	m
Water use	200	2.285	3.000	3.000	3.000	3.000	3.000	3.000	2.898	m^3/s
Water use	201	1.283	1.074	1.015	1.088	1.253	1.423	1.601	1.248	m^3/s
Channel	400	2.285	3.090	3.608	3.987	4.289	4.480	4.641	3.769	m^3/s
Channel	401	2.285	3.000	3.000	3.000	3.000	3.000	3.000	2.898	m^3/s
Channel	402	0.000	0.090	0.608	0.987	1.289	1.480	1.641	0.871	m^3/s
Channel	403	0.783	0.574	0.515	0.588	0.753	0.923	1.101	0.748	m^3/s
Channel	404	1.283	1.074	1.015	1.088	1.253	1.423	1.601	1.248	m^3/s
Channel	405	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	m^3/s
Objective function		366.012	353.023	347.819	355.752	372.818	390.333	408.598	372.233	

Table 11. Solution to Test Problem 6—Limited Outflow Capacity, Reservoir 2 Value=20

Component type	Component number	Iteration 1	Iteration 2	Iteration 3	Iteration 4	Iteration 5	Units
Reservoir	1	1,665.226	1,663.954	1,663.677	1,663.663	1,663.663	m
Reservoir	2	1,662.635	1,664.109	1,664.378	1,664.390	1,664.390	m
Water use	200	0.000	0.000	0.000	0.000	0.000	m^3/s
Water use	201	3.000	3.000	3.000	3.000	3.000	m^3/s
Channel	400	1.820	3.285	3.577	3.591	3.591	m^3/s
Channel	401	0.000	0.000	0.000	0.000	0.000	m^3/s
Channel	402	1.820	3.285	3.577	3.591	3.591	m^3/s
Channel	403	0.000	0.000	0.000	0.000	0.000	m^3/s
Channel	404	3.000	3.000	3.000	3.000	3.000	m^3/s
Channel	405	1.000	1.000	1.000	1.000	1.000	m^3/s

this case is shown in Table 11. It is reasonable to assume that reservoir 1 was not built to always bypass supply to irrigation block 200, especially when there is no apparent need for it. Consequently, this value system is not a general solution here, since it tends to introduce its own problems for a range of possible hydrologic conditions.

To summarize, the value system used in Test Problem 1 without flow restrictions works well both with the NFA and with the full LP application, where the full LP application includes the piecewise segmentation of the outflow versus storage curve that cannot be included in the NFA problem formulation defined by Eqs. (1)–(3). However, when outflow restrictions are introduced, the full LP application still works with the same value system used for Test Problem 1, while the iterative scheme within the NFA fails to deliver sensible solutions for a number of tests demonstrated in this paper. A cost factor-based priority system should describe an operating policy that should be the same for all simulated time intervals and choice of flow constraints. It should not depend on the hydrologic conditions. That is clearly not the case in the test problems presented here.

Conclusions and Recommendations

This paper examines the limitations of network flow algorithms to address nonnetwork constraints using an iterative approach. Although the paper by no means includes all instances where an iterative scheme may fail, it can be generally concluded that any flow path restrictions that are updated through iterative calls of the NFA solver may fail to deliver reasonable solutions. The limitations are demonstrated using numerical examples with sufficient input data to allow independent verification. The numerical examples presented in this paper include a simple system with two reservoirs in series, which is an elementary configuration common to most water resources systems. Another possible failure of the NFA-based iterative schemes is an example of a single reservoir with multiple outflows, where flow limits in some of those outflows are governed by the outflow versus elevation curve. It has been documented that such configuration can cause failure for both the NFA solver and in some instances for full blown LP solvers (Ilich 2008). It is believed that most water resources systems contain reservoirs in sequence, reservoirs with multiple outflows, or a combination of both. This paper questions the fitness of the final solution to which NFA models may converge for two or more reservoirs in series, while the other referenced publication (Ilich 2008) examines the use of NFA solvers on reservoirs with multiple outflows. Together, the two publications question the wisdom of using iterative schemes used with NFA models to handle nonnetwork flow constraints. This issue deserves attention since many NFA models with built-in iterative schemes are still actively in use by various water resources practitioners around the world.

Acknowledgments

The Water Resources Management Model (WRMM) used as the benchmark NFA-based model is a public domain modeling tool owned and maintained by Alberta Environment in Canada. Sincere thanks go to George Kuczera who independently verified the results of the test problems presented here, and also to an anonymous reviewer and the journal editor for their useful suggestions.

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