Hydrological Sciences Journal

ISSN: (Print) (Online) Journal homepage: www.tandfonline.com/journals/thsj20

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HYDROLOGICAL SCIENCES JOURNAL
https://doi.org/10.1080/02626667.2024.2394640

REVIEW

Practical applicability of mathematical optimization for reservoir operation and river basin management: a state-of-the-art review

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ABSTRACT
The sheer number of publications that deal with the topic of optimizing the management of river basins has grown exponentially since the early 1980s, and this growth is still on the rise. Despite this, the practical actions of most reservoir operators are still based on their gut feelings, or at best on straightforward rules that did not originate from rigorous scientific studies but are rather the result of the operator's experience or simple spreadsheet calculations. Many publications have already pointed out the gap between theory and practice over the past few decades; however, none have so far offered clear guidelines on how to overcome this gap. This paper presents an extensive literature review to examine potential reasons for this gap. In addition to this, a numerical test problem demonstrates a novel way of using linear programming for constructing Pareto-optimal solutions for a large class of multi-objective optimization problems.

ARTICLE HISTORY
Received 26 March 2024
Accepted 7 August 2024

EDITOR
A. Castellarin

ASSOCIATE EDITOR
Y. Zhou

KEYWORDS
optimization methods; optimization constraints; reservoir rule curves; optimal demand hedging

1 Introduction

1.1 Challenges in river basin management

The complexity of managing modern river basins has led to various applications of systems analysis techniques, which were defined by Rogers and Fiering (1986, p. 146S) as a "set of mathematical planning and design techniques which at least include some formal optimization procedure." In this context, optimization is understood as a process of selecting one or more solutions that have superior qualities in terms of management objectives compared to other arbitrarily selected solutions, while they are also feasible concerning the physical and operational constraints. In recent decades, much of the discussion has focused on the issue of defining objectives, while the treatment of constraints seems to have taken a back seat.

In their discussion about the future of the science of water resources systems analysis, Brown *et al.* (2015) made a distinction between its use for (a) planning studies related to water policy; (b) addressing trade-offs among multiple objectives; (c) water resources operations; and (d) the water distribution systems related primarily to design and operation of pressurized water supply networks, which are outside of the scope of this paper. In essence, the use of a scientific approach should help generate better river basin management plans and assist with their implementation in real time, assuming the required input data are also available in real time. This should emphasize the link between (a) and (c), since planning studies are *de facto* conducted to gain insight into river basin operation under various changing conditions that may involve structural changes or changes in hydrological conditions.

The term river basin management is often interchanged with terms like reservoir management or reservoir operation, due to a sizeable overlap in their functionality. Indeed, reservoirs are indispensable for sustainable river basin management, since they store water during high runoff to help reduce or eliminate deficits at times of shortage. Without reservoirs, the notion of river basin management may be reduced to controlling sediment erosion or the concentration of pollutants at the source, but the natural flow regime with its wild swings between floods and droughts would remain unchanged. Reservoirs should be managed optimally for all of their intended and often conflicting goals, which may include water supply for domestic or industrial purposes, irrigation, environmental maintenance flows, or power generation. In general, the operational goals are to help minimize flood damage and reduce the duration and magnitude of water shortages. These goals require resolutions at different time scales that range from hourly for flood events up to monthly for drought management, requiring different problem definitions and solution strategies (Labadie 2004, Rani and Moreira 2010, Azad *et al.* 2020). Therefore, improved operation of the existing reservoirs is as important as is optimal design of new reservoirs, and advancing the state of the art in the area of reservoir operation represents an area of active research (Dobson *et al.* 2019). This is supported by the fact that there is presently a large number of publications that deal with the topic of reservoir operation that has grown exponentially since the early 1980s, and this growth is still persistent, as shown in Fig. 1.

The sheer volume of the models investigated by the researchers poses the question of their applicability in river basin management and operation. If used as planning tools, they should help generate reservoir operating rules and water

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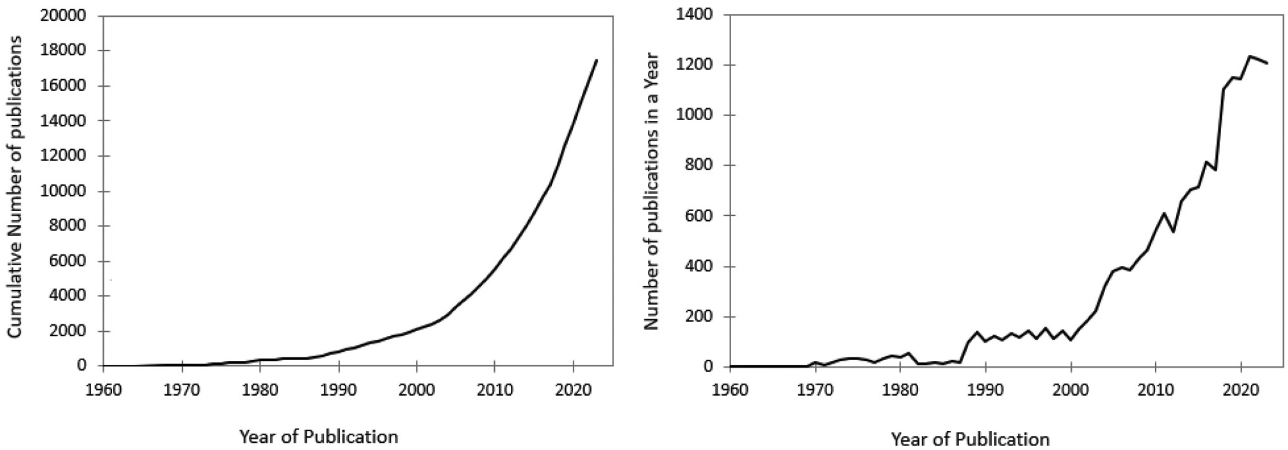

Figure 1. The number of publications with "reservoir operation" search terms in the title, abstract, or keywords. Obtained from the SCOPUS database on 29 December 2023.

rationing policies as integral parts of river basin plans. If used as operational tools in combination with runoff forecasts, they should suggest the best set of reservoir releases subject to the basin conditions and runoff forecast. Given the above, the principal question addressed in this paper can be formulated as follows: "How do the numerous publications related to reservoir optimization help the basin managers and reservoir operators?" The paper proceeds as follows: Section 2 gives a historical background related to the use of optimization in water resources, with a particular emphasis on the problem definition and relevant constraints, including some observations related to the widespread use of multi-objective optimization. Section 3 explains the current operators' practices, while Section 4 provides a statistical summary of a survey of selected papers, particularly concerning the inclusion of the constraints that are considered important based on the previously outlined problem definition. As an example of the importance of one of the highlighted constraints in the survey in the previous section, Section 5 provides a numerical example that demonstrates the importance of the reservoir outflow constraints that are equally applicable to single and multiple objective optimization, while Section 6 gives conclusions and recommendations.

2 Problem definitions and historical background

We define the term "model" as a representation of reality. In the case of river basin models, we typically refer to a mathematical representation of reality encapsulated in the form of mathematical algorithms and a computer program that mimics decision making processes, acting as a "crystal ball on the table" for river basin managers and reservoir operators. Since the real world is inherently very complex, models invariably involve some level of simplification of reality. However, a properly designed model should have a sufficient level of complexity to include all important aspects of reality, without being cluttered with too many unnecessary details, which may lead the modelling practitioners to a situation where they "cannot see the forest for the trees."

The most basic distinctions among models are between *simulation* and *optimization* models. Rogers and Fiering (1986) define simulation models as "descriptive techniques" that imply the application of "what-if" rules that are triggered by the storage levels, inflows, and demands evaluated individually in each simulated time step. These models do not define the best releases over a simulated period, but are rather aimed to derive releases by following a set of prescribed rules. A historical example is the Hydrologic Engineering Centre (HEC-5) model, which was eventually renamed HEC-ResSIM (US Corps of Engineers 2024), one of the few public domain models among its several well-known commercial counterparts such as RIBASIM (DeltaRes 2024a) or Mike Hydro Basin (Danish Hydraulic Institute 2024).

Originally developed for single reservoir systems, the use of "what-if" rules became difficult to implement even for single-time-step solutions, in the case of river basins with multiple reservoirs and moderately complex network configurations. Consequently, the problem of water allocation based on prescribed rules was facilitated by the use of optimization algorithms, although they were guaranteed to find the best solutions that followed for individual time steps, without taking into account the consequences of the current time step solutions for the system performance in subsequent time steps. Representatives of these improved simulation models with built-in optimization algorithms to assist with water allocation in single time steps include MODSIM (Labadie *et al.* 1986), Water Evaluation Assessment Program (WEAP) (Yates *et al.* 2005), Resource Allocation Model (REALM) (Victoria State Government 2024) and AQUATOOL (Andreu *et al.* 1996), and they are all typically based on simplified linear programming (LP) solvers known as network flow algorithms (Bertsekas and Tseng 1988), except for the WEAP model, which uses a full LP solver that allows more versatile representation of constraints. The choice of LP was driven by several factors, such as multiple publicly available solver libraries, fast execution times and the guarantee of finding the global optimum (albeit for individual time steps), along with the fact that water rationing rules were easy to formulate as LPs.

The use of LP solvers to model a sequence of individual time step decisions is referred to as simulation-optimization modelling by some authors, such as Fayaed *et al.* (2013) in their review paper on reservoir system management techniques. They differentiate simulation-optimization models from full reservoir optimization models by their ability to find the best set of releases over a specified period T which involves multiple time steps t ($t = 1,T$), usually referred to as "multi-period" or "multiple time step" optimization, where the reservoir releases made in one time step have consequences that are felt two or more time steps later at critical downstream locations, after they have been modified by the effects of hydrological routing and additional influx from tributaries or diversions at water intake structures (Fig. 2).

The same physical system is repeated in Fig. 2 for three consecutive time steps in the left to right direction for demonstration purposes. Releases made in the first time step undergo hydrological routing transformation as they propagate downstream. This transformation should be modelled as a non-linear constraint for daily or hourly time steps when the travel time through the entire system is longer than the calculation time steps, since storage releases need time to reach the critical downstream locations. This link cannot be modelled directly with simulation models since they only model individual time steps. The effects of travel time and hydrological channel attenuation are typically ignored in most publications by authors who enthusiastically devote a lot of space to explain the heuristic solution algorithms to which they ascribe real-time optimization capabilities for managing floods or hydropower operation, without mentioning the need for proper inclusion of hydrological channel routing as constraints into optimization. A typical hydrological routing equation is:

$$Q_{t+1}^{out} = C_1 Q_t^{in} + C_2 Q_{t+1}^{in} + C_3 Q_t^{out} \quad (1)$$

While the above equation has the form of the well-known Muskingum equation, it should be noted that continuous modelling that includes a variation of channel flows cannot be represented properly using fixed routing coefficients that are determined for a single event using the Muskingum method (Ponce and Yevjevich 1978). Rather, the routing coefficients change with the change of flows which determine the travel times through a river reach. It is interesting to note that the full definition of the reservoir operation problem using the LP

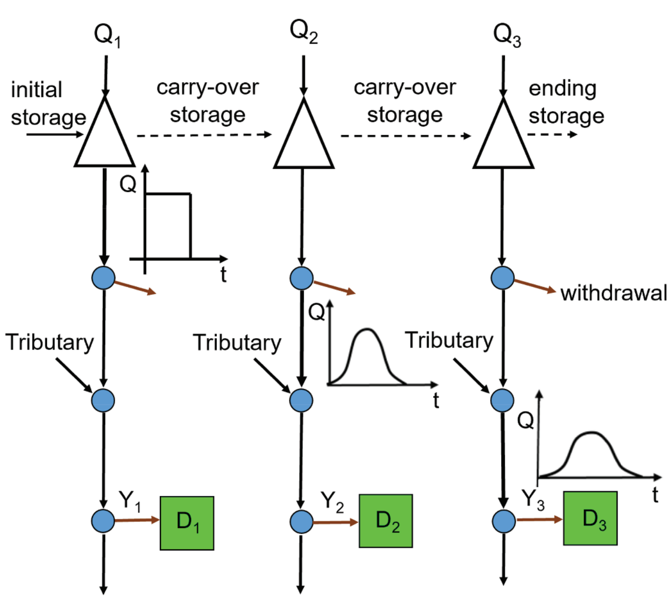

Figure 2. Schematic representation.

formulation was given five decades ago by Windsor (1973), whose initial formulation was made for a single time step, and later extended to a multiple-time-step solution framework by Yazicigil *et al.* (1983) and Needham *et al.* (2000). Ignoring the hydrological routing constraints given by Equation (1) is possible when conducting a river basin planning study that is focused on drought management, where a weekly time step may justify the steady-state assumptions if the travel time along the entire river basin is on the order of 4 to 5 days. This usually restricts the size of the basins being modelled to less than 400 km in total length. Despite the huge volume of papers that cover heuristics search engines and dynamic programming (DP) (Macian-Sorribes and Pulido-Velazquez 2020, Krit *et al.* 2023), the only commercially available models that were identified as capable of including channel routing as constraints in multiple time step optimization are RTC (DeltaRes 2024b); RTO (Kisters 2024); and Web Basin Management (WEB.BM) (Ilich 2021; Ilich and Basistha 2024), and these models rely on classical LP or non-LP solution algorithms. Hydrological routing is required for modelling with time steps that are daily or shorter, since the size of a typical river basin involves travel time throughout the basin (i.e. time of concentration) that is usually much longer than 1 day. Real-time operation or planning studies that examine reservoir operation during floods require the use of daily (or shorter) time steps, while studies that focus on managing droughts typically use weekly, 10-daily or monthly calculation time steps and they do not require channel routing transformation.

In terms of the options related to hydrological inputs, Fayaed *et al.* (2013) provide a clear distinction between deterministic and stochastic optimization, as well as the differences between implicit and explicit stochastic optimization. The need to resort to stochastic functions is driven by the lack of reservoir inflow series of sufficient length. Implicit stochastic optimization implies the development of stochastic time series of hydrological variables, such as inflows and precipitation as representative flow series that can portray various future runoff conditions, which are then fed to the optimization models as input data. Conceptually, this solution strategy is not different from using the historical time series of inflows as input. Assuming that water allocation provides benefits to all stakeholders and that the specified period of known inflows T covers the entire simulated period, the objective function can be generally defined as:

$$max \sum_{t=1}^{T} \beta(s_t, r_t) \quad (2)$$

where β represents the benefit function associated with the ending storage s_t and the end of each simulated time step t and the average regulated flows r_t that represents allocation released to various stakeholders in each time step t. The above maximization problem is subject to the following constraints.

Mass balance constraints for every node in the network have the following general form:

$$s_{t+1} = s_t + \sum_{i=1}^{m} q_{i,t} - \sum_{j=1}^{n} r_{j,t} - \sum_{k=1}^{l} loss_{k,t} \quad (3)$$

where q_i and r_j represent average inflow and outflow from a node via their respective channels i and j, while the loss term may represent losses with index k that may be related to losses due to net evaporation or seepage in time step t. If there is no storage for a particular junction or diversion node, the above equation is still applicable except that the storage terms s_t and s_{t+1} are set to zero. Storage constraints typically require definitions of the minimum and maximum operating levels which can also be temporal variables:

$$s_{min,t} \leq s_t \leq s_{max,t} \tag{4}$$

where the minimum and maximum storage may be defined in general as a function of time throughout the year as operational constraints that require modification of the minimum and maximum allowable limits.

Losses can be related to canal losses or losses to net evaporation from the surface water area of reservoirs, which should be calculated using the following formula:

$$loss_t = NE_t = (e_t - p_t)\frac{(A_t + A_{t+1})}{2} \tag{5}$$

where e_t and p_t are respectively evaporation and precipitation on the surface area A of the reservoir over a time step t, while A_t and A_{t+1} are the water surface areas at the start and the end of time step t. Net evaporation is the difference between evaporation and precipitation in time step t. This implies that in some time intervals during rainy seasons the losses due to net evaporation can be negative, since the value of precipitation may exceed the value of evaporation. Although correctly defined, the above loss function is rarely used in this form. It is usually either completely ignored, or if it is used it is defined as evaporation, not as net evaporation, thus ignoring the precipitation directly on the water surface area.

2.1 Reservoir release constraint

In most published studies flow releases r_t are restricted by the constant upper levels determined either by the target water demands or by the installed capacity of the turbines if the releases are made to generate hydropower, using Equation (6):

$$r_{min,t} \leq r_t \leq r_{max,t} \tag{6}$$

where the lower release levels may be required as the minimum maintenance flows. The above relationship is the most common way of modelling the upper limits on channel flows, which is problematic for reservoir outflows since it ignores the physical relationship between the maximum outflow and the available storage given by the outflow vs elevation curves that can be associated with bottom outlets, spillways or turbines. In each instance the maximum flow $r_{max,t}$ may be reduced from its constant value to a value that corresponds to the average storage over the time step t based on the outflow vs storage relationship which exists for all physical outlet structures. The correct formulation of the above constraint is:

$$min\left\{Q_{max}\left(\frac{s_t + s_{t+1}}{2}\right), r_{min,t}\right\} \leq r_t \leq min\left\{Q_{max}\left(\frac{s_t + s_{t+1}}{2}\right), r_{max,t}\right\} \tag{7}$$

where the function $Q_{max}(s)$ refers to the maximum outflow as a function of the available storage s based on the outflow vs storage curve. This constraint is almost routinely ignored when modelling hydropower generation, implying that the assumed outflow through the turbines can always reach the installed turbine flow capacity regardless of the storage levels, which may lead to gross errors in time intervals when the reservoir storage is low. The outflow limit constraints for the bottom outlet and spillway are demonstrated in Fig. 3 using the example of Dickson Dam in the Province of Alberta, Canada. The use of LP solvers requires piece-wise linearization of the maximum outflow vs storage function, and also the introduction of binary variables, as detailed by Needham *et al.* (2000) and Ilich (2008), who defined the need to use mixed-integer linear programs by advanced modelling tools such as WEAP (Yates *et al.* 2005), OASIS (Randall *et al.* 1997) and RiverWare (Zagona *et al.* 2001), or WEB.BM (Ilich 2021). The importance of proper modelling of the outflow constraints as a dynamic function of storage is demonstrated in the numerical example provided in Section 5.

The curve in Fig. 3 shows a very low outflow capacity of the bottom outlet compared to the spillway. If the modellers ignore this relationship, their models would allow the release of any amount of flow regardless of the storage level. For example, if the modelling goal was to manage a large incoming flood, the model could release 900 m^3/s (which corresponds to the full bank downstream channel capacity) for 2 or 3 days before the arrival of the incoming flood peak, thus reducing the storage significantly and creating comfortable extra storage for flood protection. Yet it is obvious from the graph in Fig. 3 that such an operation would not be physically possible, given that the maximum release from storage is severely restricted if the level drops below 941 m. Also, when reservoir levels are below 930 m, the bottom outlet capacity may be insufficient to meet downstream demands. Ignoring the outflow capacity constraint would allow the reservoir to route the sufficient from other upstream reservoirs to satisfy water demands which may exceed the outflow capacity, without ensuring that there is sufficient storage volume to make the desired releases physically possible. Hence, addressing constraints of this kind is important to make sure that the model solutions are meaningful and acceptable to the reservoir operators. It should also be noted that this constraint is valid for any length of time step, from hourly to monthly.

Some other operational constraints can and should be modelled as hard constraints, since their violation would have legal implications. Those include for example the annual diversion volume licence limits, which cannot be exceeded

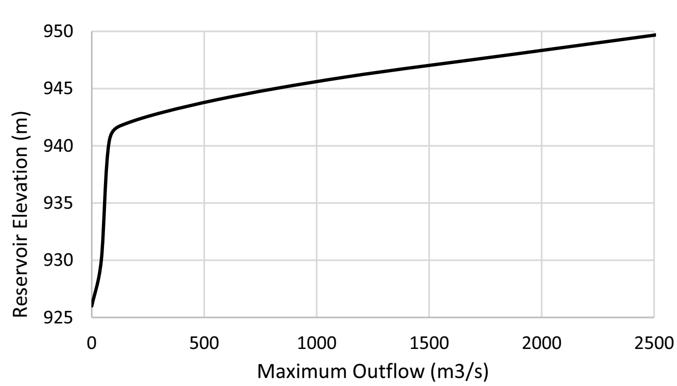

Figure 3. Dickson Dam elevation vs outflow (Alberta, Canada).

during an irrigation season, or the apportionment agreements between bordering states or provinces, which are usually associated with a combination of instantaneous minimum flow at the border crossing and the target volume requirement that needs to be passed to the downstream jurisdiction over a specified time period.

A computer modelling tool that can successfully solve the above reservoir optimization problem with all relevant real-world constraints would certainly be of assistance to reservoir operators, especially if there was also a reliable runoff forecasting tool that could provide real-time runoff forecasts as updated daily model inputs. This should be the ultimate goal of the model development. Given that perfect runoff forecasts for periods of up to 5 days are not yet available, the focus of the modelling community should be to improve the planning studies to develop reservoir operating guidelines that should aid the reservoir operators.

2.2 Disadvantages of using monthly time steps

Most publications available in the literature rely on the use of monthly calculation time steps, without making an effort to evaluate the disadvantages of this approach. The monthly hydrograph consists of the mean daily flows averaged over a month. Monthly averages are close to daily flows only during dry seasons, which are characterized by low fluctuation of daily flows. However, using monthly calculations during high-flow seasons leads to gross misrepresentation of daily flows, as can be seen in Fig. 4 which compares daily and monthly natural flows at the Smoky River in Northern Alberta (Water Survey of Canada 2024).

It is well understood that the use of mean monthly flows is not suitable for studying reservoir operation during floods. Compared to monthly time steps, modelling daily time steps would certainly require different storage drawdowns to ensure minimizing unnecessary spills that bypass turbines. Studies that focus on the impacts of the time step length (daily, weekly or monthly) require comparisons of the model outputs where all other model inputs are the same except for the time step length. Such studies are missing in the literature. Ideally, for steady-state calculations, the calculation time step length should be longer than the travel time through the entire river basin by a factor of 2 or more, to ensure that most of the release from the upstream reservoirs can reach the most downstream components within the same time interval. This is often violated in many studies where daily time step is used on river basins with the total travel time through the basin of several days. Optimization models rely on a basic premise that reservoir releases are demand-driven. Since most basins have travel times longer than one day, this premise requires solving multiple time steps simultaneously for daily time steps, as shown in Fig. 1, such that the release decisions can reach intended downstream demands after more than one day of travel time. The model should therefore be able to determine both the timing of the releases and their quantities to reach the designated downstream users while accounting for all hydrological transformations along the way.

2.3 Previous review papers and the current state-of-the-art related publications

Most of the early attempts to utilize the results of optimization used monthly time steps to develop a regression that would provide forecasts of the reservoir levels at the end of the month based on starting storage and anticipated monthly inflow (Young 1967). Since regression can sometimes result in outcomes that are outside of the expected range, Karamouz and Houck (1982) introduced an additional correction factor to keep the results of the regression within an anticipated range, while Willis *et al.* (1984) developed a probability density function of optimal releases based on selected state variables and statistical analyses of the output of implicit stochastic optimization. It should be noted that these attempts relied on monthly optimization results that also ignored the reservoir outflow constraints. Attempts were also made to use artificial neural networks (ANNs) to derive operating rules from the results of stochastic optimization (Raman and Chandramouli 1996, Chandramouli and Deka 2005, Farias *et al.* 2006). In general, in both the regression and the ANN-based models the releases obtained by the optimization model are related to reservoir storage at the end of the previous (typically monthly) time steps, and inflow during the current time step. There are several issues when attempting to apply this approach in real time: (a) monthly runoff forecast is not available with sufficient certainty;

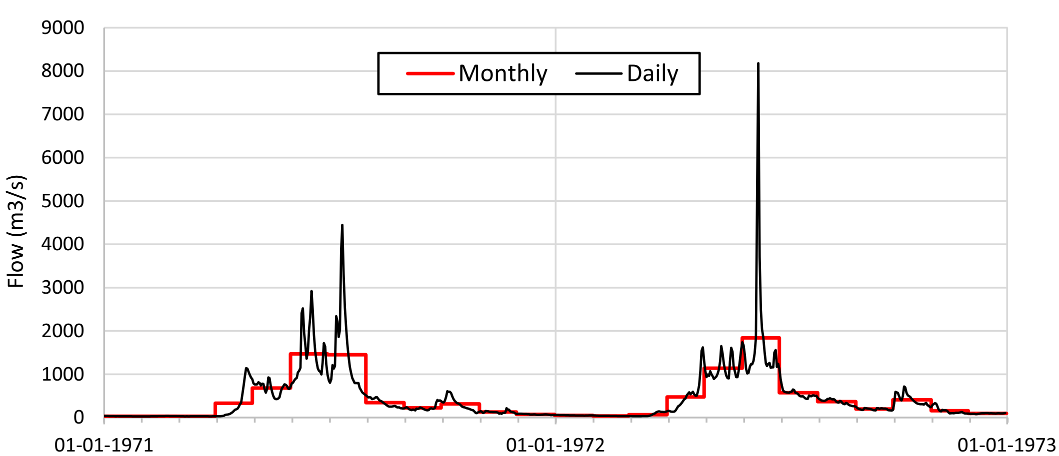

Figure 4. Monthly and daily hydrographs, Station 07GJ001, Smoky River, Alberta, Canada.

(b) monthly forecast implies fixed monthly flows, which does not correspond to reality; (c) reservoir releases determined by regression will likely not match the current downstream demands; and (d) reservoir releases determined in this way may not be physically possible due to the limitations imposed by the outflow vs elevation curve, although the constraint that defines maximum outflow as a function of storage is found in some attempts to apply adaptive neuro-fuzzy systems to reservoir operation (Mousavi *et al.* 2007, Mehta and Jain 2009), or in the attempts to introduce parametric simulation-optimization (Koutsoyiannis and Economou 2003).

While the popular models that have tens of thousands of users rely on LP (e.g. MODSIM, WEAP, RiverWare), the authors of many recent review papers hardly take any notice of them in their reviews (Kumar and Yadav 2022, Lai *et al.* 2022, Kangrang *et al.* 2023). Instead, the focus is on "popular" models based on heuristic search engines, while the modelling objectives have shifted to "the search for an optimal policy" rather than the search for the best model solution, implying a strong shift to multi-objective optimization under the pretext that the river basin managers can no longer define their management objectives, so they have to rely on the multi-objective optimization models to help them improve the understanding of their priorities. The emphasis has moved from finding the best reservoir releases for a given set of inputs to finding the solutions that are "good enough," that are non-dominated, "equally optimal" or "Pareto-optimal" with respect to two or more operational goals. The number of publications that cover multi-objective programming has been rising faster than the classical multi-purpose optimization papers, such that some recent review papers completely ignore the classical optimization techniques (Lai *et al.* 2022). This is all happening in spite of many reports that practitioners are having difficulties understanding the results and finding meaningful ways to apply them in practice (Castelvecci 2016, Quinn *et al.* 2019). A reservoir operator needs guidance on how much water to release on a daily basis. Offering 100 or more Pareto-optimal solutions instead for multi-year model runs provides a huge amount of data which is difficult to convert to a practical operating guideline. While the gap between theory and practice has existed for decades (Simonovic 1992), the increasing multitude of papers that utilize multi-objective programming has done nothing to close it. On the contrary, the principal reason for this is the introduction of the idea that reservoir operating rules are difficult to define explicitly due to conflicting objectives among various stakeholders, often driven by the fact that some objectives, such as environmental flow targets, cannot be defined using typical economic value functions, while fitting them into the existing priority chain is routinely questioned by other water users. In essence, this introduced the need to redefine the objective function by removing the explicit priority of one stakeholder over another, and rather defining objectives by using system-wide parameters, such as reliability or shortage index, vulnerability, the highest water shortage, or the frequency of water shortage as measures that are applied evenly on all types of water use (Krit *et al.* 2023). There is a prevailing attitude in the recent literature that reservoir operation is a "wicked" problem (Mamatova *et al.* 2016, Wu *et al.* 2023). However, for most real-world reservoir operators, the rules are clear, and the allocation priorities are defined by the governing water management committees which are also staffed by stakeholders. Allocation policies are based on either the legal priorities arising from the water licensing system, as is the case in North America, or a mix of economic and political objectives, and often as an agreed combination of both. In most river basins around the world, the sum of municipal and industrial water use constitutes a small fraction of the total irrigation water use. Consequently, any form of equal deficit sharing between the two would not make much sense, since shorting municipal supply would not help the irrigators in any meaningful way, while it would enrage the urban population. Furthermore, reducing flood damage should not be a priority available for a trade-off with any other objective, especially for basins that have early flood warning systems, in spite of the fact that many researchers are keen to use multi-objective optimization in an effort to find a compromise between reducing flood damage and reducing the loss of hydropower generated during floods (Moridi and Yazdi 2017). It is not clear that there has to be any loss in hydropower during floods, especially since most larger basins have flood warning systems which cause the operators to lower the reservoir storage prior to the arrival of the flood peak flow. When such pre-flood drawdown is achieved by releasing water through the turbines so as to minimize spills, it is only a matter of the available lead times and the capacity of the turbines that determines the best operation that maximizes both benefits (reduction of flood damage and maximization of hydropower), as demonstrated by Ilich and Basistha (2024) in their recent work. In addition to the questionable compromise between these goals, a large number of publications still entertain this topic, sometimes using the monthly calculation time steps to model floods, as is the case with the work of Hatamkhani *et al.* (2021), while simultaneously ignoring the important reservoir outflow constraints for hydropower, which should be modelled as a dynamic function of storage as defined by Equation (7).

Other conflicting objectives that are usually modelled in various studies may involve maximizing hydropower and maximizing irrigation; however, most reservoirs are built with one of those objectives as its primary purpose which completely dominates the secondary purpose. One issue that may justify using a multi-objective optimization approach is the environmental river maintenance flows, which have emerged as a more recent target, long after many reservoirs had been constructed. In the past, water quality studies have been conducted to determine biological minimum flows that were then used as target in-stream maintenance flows in river basin models, thus creating two steps in the modelling process which separated modelling of water quality and quantity. Multi-objective optimization has managed to creep into this area as well, postulating the importance of modelling to determine reservoir releases via multiple outlets located at different elevations. Examples of such studies are available from Karamouz *et al.* (2011) and Aghasian *et al.* (2019), although they also use monthly calculation time steps which are not adequate for representing downstream river flows, and ignore the reservoir outflow constraints, which should be of importance given multiple outflows with various outflow elevations and reservoir storage that may vary significantly during the year, thus affecting the dynamic outflow capacity of each outlet. The purpose of multi-objective programming would be to find a suitable way of fitting the priority of environmental flows among other water allocation priorities such that the final choice of priority is acceptable to all stakeholders. This case is used to compare the basic ideas related to multi-objective optimization with the regular definition of mathematical programming to try to identify the intersection between the two. Simonovic (2008) provides a more detailed coverage of multi-objective optimization. To ensure discussion, the left side of Fig. 5 shows a typical graph used to define the so-called non-dominated or Pareto-optimal solutions related to objectives 1 and 2. The short straight lines that limit the solutions on both axes show the maximum possible performance where all demands are met for one objective at the expense of the other objective. The solutions located in the middle range are not dominated by either of the two objectives, so they are referred to as a non-dominated or Pareto-optimal set. It is important to note that each solution in this set consists of a time series of regulated flows and reservoir levels over a multi-year simulated period. The performance evaluation for each non-dominated solution presented in Fig. 5 is therefore a formidable task, and yet the proponents of multi-objective programming advocate evaluation of a selected set of those solutions, although there are no clear guidelines on how such a set should be selected.

Since the heuristic algorithms progress in their search by recombination of a group of solutions, they are more suitable for multi-objective programming. The schematic presentation of selected non-dominated solutions can be compared with the case of having multiple optimal solutions in the classical definition of mathematical programming displayed on the right side of Fig. 5. The objective function for a linear program with two variables is defined as:

$$\max Z = C_1X_1 + C_2X_2 \tag{8}$$

where the parameters C_i are weight factors that represent operational priorities. For instances where $C_2 > C_1$, the optimal solution will be selected at the intersection of line a with the feasible region shown on the right side of Fig. 5, while for instances where $C_1 > C_2$, the best solution will be selected at the intersection of line b with the feasible region. The case where $C_1 = C_2$ creates a solution where any point on the edge of the feasible region that overlaps with line c is equally optimal. Within the operations research community at large, this would be considered a "poorly defined optimization problem" since it has an infinite number of equally optimal solutions. In contrast, in the community of water resources practitioners, this problem definition is celebrated through the practice of multi-objective optimization. Although LP derives only a single solution, some researchers, like Rozos (2019), went through painstaking sensitivity analyses to determine the values of weight factors C_1 and C_2 with minute differences until they found solutions that are different in terms of the values of X_1 and X_2, by using two different network flow models (MODSIM and HEC-ResPRM) such that the values of their objective functions are the same on the first three decimals, thus trying to mimic the instance of finding different solutions that are equally optimal. It should be noted, however, that it is possible to fix a desirable solution out of many equally optimal solutions by introducing additional constraints in the following form:

$$\frac{X_1}{U_1} = \alpha\frac{X_2}{U_2} \tag{9}$$

where U_1 and U_2 are the upper bounds (water demands). If factor $\alpha = 1$, this constraint ensures an equal relative deficit between the two objectives to maximize X_1 and X_2, thus ensuring that both objectives receive equitable allocation by making their relative deficits equal. Other values of factor α would result in different sharing policies between X_1 and X_2, for example 40/60, 30/70, or 60/40 and 70/30, expressed as a percentage of the total demand, where U_1 and U_2 represent the total demands.

With a few exceptions, such as Rozos (2019), the use of LP in multi-objective optimization seems to be of no interest to researchers, precisely because it ends up finding a single solution. Rather, most multi-objective publications rely on the use of heuristic solvers. There is currently a plethora of (meta)heuristic optimizers available, and none of them has been proven superior over the others, which motivates the scientific community to continuously evaluate their performance and put in efforts to enhance them (Maier *et al.* 2014). These efforts have gone in two directions: to develop combinations of several optimization

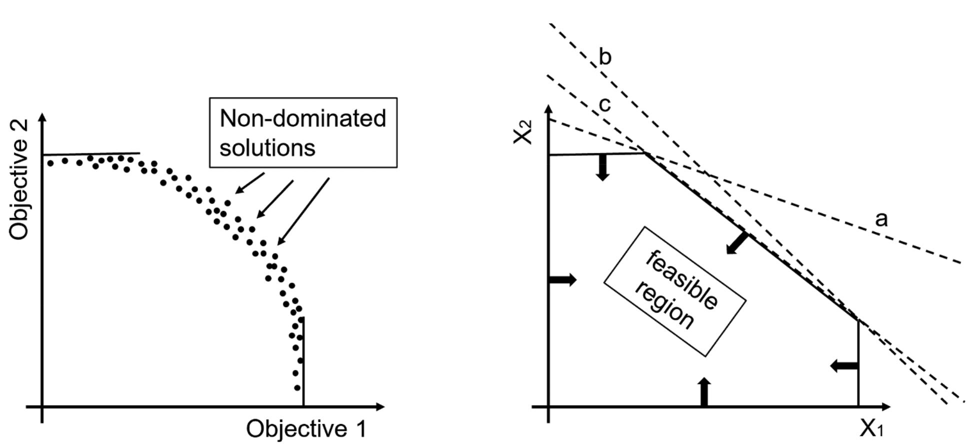

Figure 5. Conceptual comparison of multi-objective and classical optimization.

algorithms that complement each other (Beiranvand and Ashofteh 2023), or to develop algorithms based on adaptive operator selection (Reed and Hadka 2015), where the level of an operator application (i.e. the share of the population being optimized) is allocated to each algorithm separately depending on its performance in previous generations, as proposed by Hadka and Reed (2015). Most (meta)heuristic optimization algorithms can be implemented for both single- and multi-objective problems (Dobson *et al.* 2019). The multi-objective problems can be solved either by converting them into single-objective where some objectives are treated as soft constraints, or by simultaneous multi-objective optimization that yields numerous non-dominated solutions. As for the former, it can be achieved by scaling and aggregating multiple non-commensurable objectives into a single objective (e.g. by implementing a weighting scheme), by monetizing potential benefits from the reservoir operation, or by implementing goal programming techniques that reduce multi-objective into a series of single-objective optimizations (Wu *et al.* 2023). Many researchers acknowledge that multi-dimensional Pareto fronts are rather challenging to communicate to stakeholders (Wu *et al.* 2023). Therefore, a final solution is adopted by further analysing several Pareto-optimal ones. While there is no doubt that this approach has gained popularity in academic circles (Giuliani *et al.* 2014, Beiranvand and Ashofteh 2023), it is difficult to find examples of the application of these results in real time among reservoir operators. The rise of multi-objective optimization vs multi-purpose optimization has also shifted the scientific community in a new direction, where every stakeholder is treated as equal (the basic assumption being that they cannot agree on mutually acceptable water rationing policies), while the existing water licensing systems as well as the existing operating priorities in many river basins already have established priorities that are unlikely to change during the lifetime of the existing infrastructure.

3 Links between the results of optimization and the current practices of basin managers

Reservoir operation is currently driven by the judgement of the operators and the use of rule curves where available. Understanding of the rule curve concept may vary among water management agencies and reservoir operators, which justifies a quick review of its origin and purpose.

According to Dobson *et al.* (2019), reservoirs can be operated based on the standard operating policy (SOP), based on rule curves, or by using real-time optimization. The standard operating policy proposed by Bower *et al.* (1962) implies that all users' demands are met provided there is a sufficient amount of water in the reservoir, without taking into consideration future supply/demand conditions, i.e. without hedging of water demands that would reduce current supplies to reduce future deficits (Neelakantan and Sasireka 2015). As such, the SOP often results in premature emptying of the reservoirs (Spiliotis *et al.* 2016). This led to the development of rule curves, which typically consist of the upper and lower curve (Fig. 6). The upper rule curve, also referred to as the flood-limited water level in some publications (Jain *et al.* 2023), is typically shaped by the probable maximum flood studies and it involves

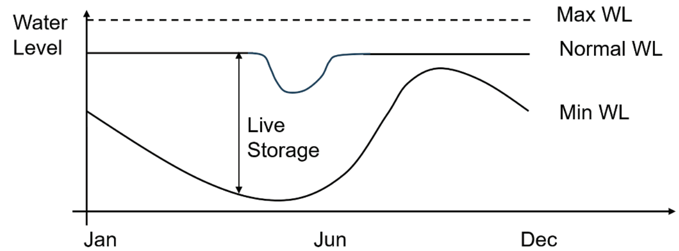

Figure 6. Schematic representation of the upper (normal WL) and lower (min WL) rule curve.

mandatory drawdown during the time of the year when large floods are likely to occur, to increase the flood storage zone and prevent overtopping of the dam (El Harraki *et al.* 2021). To avoid shortages that can occur by following the SOP during dry periods, Revelle *et al.* (1969) proposed the lower rule curve that defines the trajectory of minimum storage levels in the dry season, thus making sure that storage depletion is gradual, and storage is not empty before the end of the dry season. The actual reservoir operation can take any "live storage" level between the lower and the upper rule curves based on the operators' releases.

Although popular among practitioners, rule curves cannot guarantee optimal reservoir operation. The shape of the upper rule curve is usually fixed, while it should be different for each incoming flood that could be determined if a perfect short-term runoff forecast were available. On the other hand, the lower rule curve assumes that storage is always full at the end of each wet season, which may not be the case in dry years, thus making it impossible to follow the lower rule curve from the end of the wet season (Ilich 2023a). The shapes of the lower and upper curves typically remain the same, despite different hydrological regimes that may vary significantly from year to year. The refinement of the rule curve concept was significantly expanded with the introduction of the idea of demand hedging (Draper and Lund 2004), where the rate of storage depletion could be controlled by modifying the target demands to slow the loss of storage in dry years (Tu *et al.* 2003, Spiliotis *et al.* 2016, Ilich 2023b), by introducing operating zones associated with various levels of demand reduction (or hedging). These modifications imply a set of water rationing measures that are triggered when certain water threshold levels are reached (Garrote *et al.* 2023). These reductions are applied to prevent severe long-term shortages that would otherwise occur during dry periods (You and Cai 2008). Demand hedging is commonly applied to irrigation, since it is typically the largest consumptive water user in most river basins.

The hedging rules are identified by employing optimization methods, that range from LP to non-linear or dynamic programming, heuristic algorithms or hybrid optimization methods (Spiliotis *et al.* 2016), with various resulting mathematical formulations of these rules (Neelakantan and Sasireka 2015). The identification of optimal hedging rules represents an area of active research in itself (You and Cai 2008, Neelakantan and Sasireka 2015, Chong *et al.* 2021, Ehteram *et al.* 2021, El Harraki *et al.* 2021, Tu *et al.* 2022, Anvari *et al.* 2023, Ji *et al.* 2023, Thiha *et al.* 2023). Recent work by Ilich (2023a) provides mathematical proof that it is possible to simultaneously find both the optimal rule curve and the optimal level of demand hedging by adding the so-called "equal deficit constraints," which are defined as:

$$\frac{Y_t}{D_t} = \frac{Y_{t+1}}{D_{t+1}} \quad t = 0, \; T-1 \tag{9}$$

where Y_t and D_t represent water supply and water demand in time step t, respectively. This constraint is added as a constraint to the optimization problem to find optimal allocation over multiple time steps for the entire period T by taking into account all relevant reservoir outflow constraints, net evaporation, and mutual interaction of reservoirs in a multi-reservoir operating environment. The above constraint ensures the minimum required hedging as part of the model solution, subject to the starting storage levels, available reservoir inflows and water demands throughout T consecutive time steps. The optimal level of demand hedging and the best rule curve for each simulated year can thus be developed for a large number of hypothetical hydrological years using implicit stochastic optimization. These solutions are then analysed statistically using various techniques to help create rule curves, which provide target storage levels for subsequent time intervals for any starting storage, by matching the existing database of perfect solutions with the current conditions in the field.

There is little doubt that reservoir operators need guidance for the simultaneous management of storage releases and water rationing of downstream demands. Macian-Sorribes and Pulido-Velazquez (2020) provide a review of techniques that have been used to analyse the results of optimization models to create guidance rules for reservoir operators, ranging from regression, data mining, or the use of other artificial intelligence (AI) techniques such as ANNs and decision trees. Still, the optimality of model solutions should be based on selecting the appropriate time step, which would typically exclude monthly time steps due to considerations in Section 2.2, as well as to make sure the solutions take into account important physical constraints on maximum outflows through the bottom outlets and turbines as a function of storage. Publications that comply with the correct selection of time steps and the inclusion of the necessary reservoir outflow constraints are rare, yet only such publications provide a realistic possibility of deriving optimal solutions from which efficient reservoir operating rules could be inferred. Due to a small fraction of reservoir optimization papers that provide derivation of practical reservoir operating rules, it can be concluded that most existing operating rules are still based on the use of simulation models, or sometimes mere spreadsheet calculations. This invariably poses a question on the utility and usefulness of the research efforts to develop powerful modelling tools. Various engineering departments conduct these studies, and since engineering is applied science, it is fair to ask: Why are there so few practical applications of this research?

4 Statistical analyses of the surveyed papers in terms of handling constraints and delivering practical operating rules

The preliminary list of papers to be included in this study was obtained from the Scopus database (https://www.scopus.com/) via a search for the keywords "reservoir operation," "optimization" and "real-time" (or variants thereof), which were sought in the titles, abstracts and keywords of the documents in the Scopus database. After the preliminary screening of these documents based on their relevance to this paper, 197 of them were selected for a more detailed review according to the approach elaborated in the following section, including several that were added manually since they were relevant but were not picked up by the search function. The results of optimization can be used either as (a) input into additional analyses that should result in the creation of reservoir operating rules; or (b) real-time operational tools.

The real-time operation applicability is still somewhat theoretical, since it assumes that runoff forecasting tools with sufficient forecasting skills are available and used as input into optimization models in real time. This combination of runoff forecast and optimization is referred to as model predictive control (MPC) in some publications (Macian-Sorribes and Pulido-Velazquez 2020). Although accepted on a conceptual basis, it is difficult to infer whether the MPC results are genuinely implemented by reservoir operators, since these details are usually not explicit in publications and the forecast reliability is uncertain. Jain *et al.* (2023) provide a review of papers related to the application of multi-reservoir models for flood operation. They list 26 publications that are used as real-time operational tools, but one has to wonder how this is achieved, given that only one of the referenced papers (Prakash *et al.* 2015) takes into account hydrological channel routing based on the Muskingum method that uses fixed routing coefficients calibrated individually for each historical flood. Also, Prakash *et al.* (2015) go out of their way to formulate six objectives for reservoir operation during floods, some of which are questionable. While the objective to keep the downstream channel flow at or below the full-bank capacity is self-evident for all flood management studies, they add several additional objectives that can only cause unnecessary confusion, such as the objective to maximize the difference between the peak reservoir inflow and peak outflow during the flood event. This function would be maximized if the reservoir outflow is kept at zero during the flood, which would neither make sense nor be physically possible. However, the downstream flood damage for zero outflow would be the same as for the outflow at the full-bank channel capacity, since both outflows would result in zero flood damage. A similar propensity to unnecessarily complicate the objective function by introducing often dubious multiple objectives which are difficult to reconcile or even explain to reservoir operators is often present in other papers (Quinn *et al.* 2019, Krit *et al.* 2023). Since the multi-objective approach is in vogue, most authors feel that it is important to follow the trend by supporting the popular approach. For example, Krit *et al.* (2023) provide six objective functions expressed as the statistical evaluation of model results, of which three (the shortage index, the average water shortage, and the total square deficit) essentially represent the same composite measure of water supply deficits.

Based on the topics presented in this paper, a comprehensive assessment of optimal reservoir operation should include the proper length of the simulated time step, along with the proper inclusion of the outflow vs storage relationship and net evaporation. The Scopus search for relevant papers short-listed 197 papers that have been evaluated using several criteria presented below. The list of references of the 197 selected papers can be downloaded from Ilich and Todorović (2024), while the relevant findings are shown in Figs. 7 and 8.

Regarding the reservoir outflow limits, 53% of all publications model them as constants, while 22% model them as temporal variables associated with the seasonal variation of downstream water demands. Only 8% of all publications included the limits determined as the minimum outflow capacity (based on the available storage and the accompanying elevation vs outflow curve) and the target demands. As many as 17% of all publications did not even include any reference to the reservoir outflows. Consequently, in 92% of all surveyed publications the model results may be very different from those that would be feasible, due to ignoring the important hydraulic outflow constraints. This is demonstrated by the numerical example in Section 5. Regarding the time step lengths, monthly time steps were used in 48% of the studies, and also in about half of the studies with variable time step length (6% of the total), a small subsection of the surveyed papers where the reservoir operating rules were developed using monthly time steps and then implemented into the model with daily time steps. When it comes to hourly time steps, they generally include any lengths shorter than a day, such as 3 or 6 h. The surveyed publications that used daily or hourly time steps account for a total of 29% of all publications. However, only one in three of these papers take into account channel routing, as attested by Fig. 8, which shows that only 10% of all surveyed publications take into account channel routing. This may be justified in instances where a single reservoir is modelled without any downstream components; however, such studies are of limited value, since most river basins have multiple reservoirs.

It is also disappointing to see that only 18% of all publications include evaporation in the reservoir water balance equation, and most of them include only evaporation, rather than net evaporation (evaporation minus precipitation on the water surface area of the reservoir). About 58% of all publications flatly ignore evaporation, while the authors of 23% of the surveyed publications did not find it worth mentioning at all. The use of heuristic solvers is typically associated with the use of simplistic problem constraints. The result is that a large percentage of fixed reservoir outflow constraints in Fig. 7 (53%) are typically associated with the widespread use of

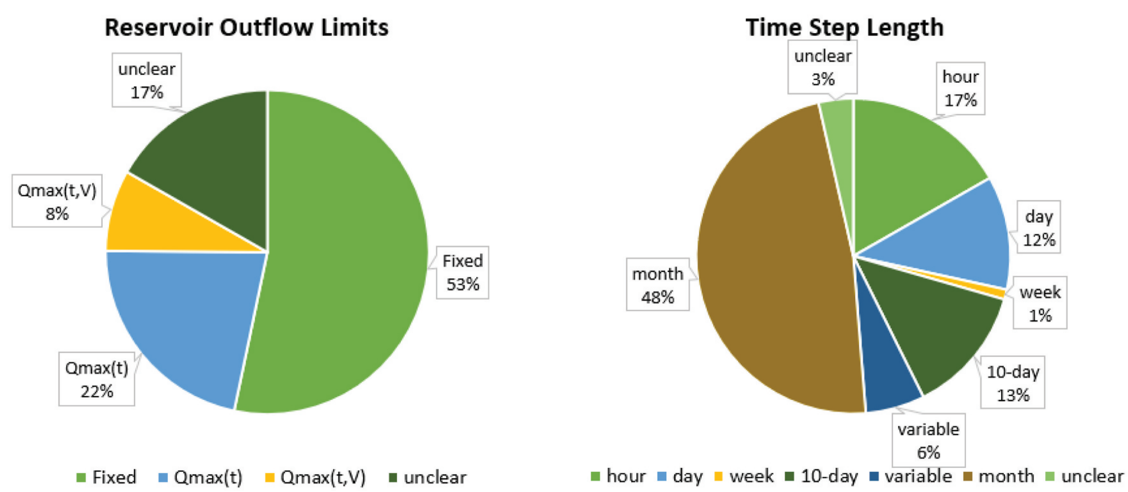

Figure 7. Reservoir outflow limits and time step length among the surveyed publications.

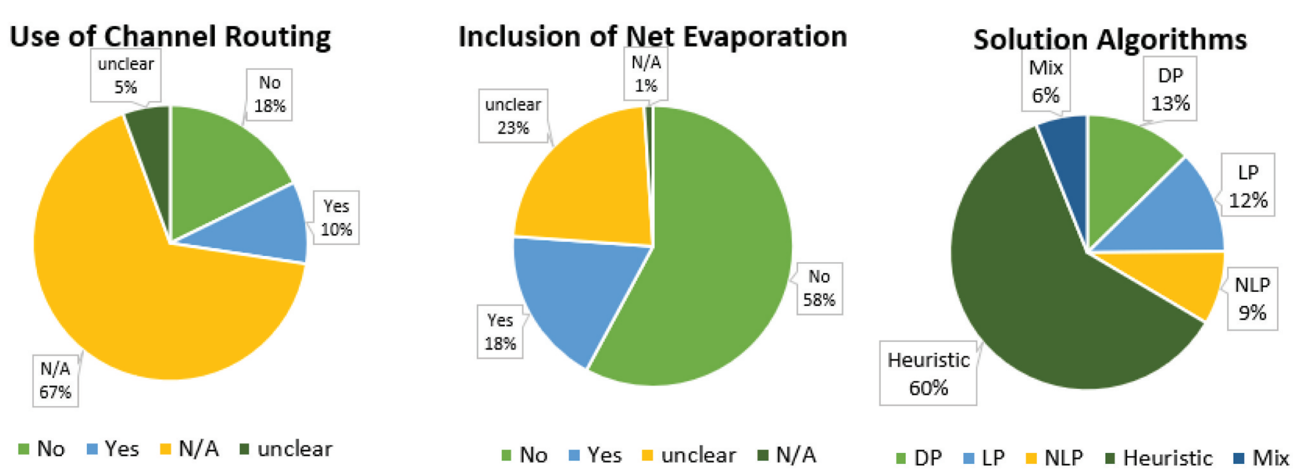

Figure 8. Classification of channel routing, net evaporation and solution algorithms.

heuristic solvers in Fig. 8 (60%). The reason for this is likely the need to add additional penalty terms to the objective function to ensure feasibility. These terms complicate the convergence to high-quality solutions and add significantly to the computational burden. Most papers that feature heuristic solvers demonstrate their use on relatively simple problems with one or two reservoirs and fewer than 10 variables, which is very small compared to the size and complexity of typical modern water resources systems. This pales in comparison with the achievements documented by papers that use LP or NLP solvers that are also used as solution engines for several known commercial tools such as WEAP, MODSIM, RiverWare, OASIS or WEB.BM. Despite all of the above, heuristic solvers are viewed favourably by a large number of researchers, whose funding is typically provided by research grants that are usually not tied to the successful application of their research by reservoir operators in the field or by water management agencies. This disconnect is also visible in Table 1.

While the aforementioned disconnect between academia and practitioners has been known for decades, this survey shows that the gap has not narrowed over time, contrary to general expectations due to the emergence of technology in the information age. The importance of the inclusion of the proper reservoir outflow constraints is discussed in Section 5.

5 Numerical example

The following numerical example demonstrates the significance of taking into account the outflow vs elevation function, as opposed to ignoring it, along with demonstrating a novel use of LP for generating Pareto-optimal solutions. The system schematic is shown in Fig. 9. There is one reservoir, one diversion canal from the reservoir for irrigation supply, and one tributary that contributes flows into a downstream river reach that has environmental flow (E-flow) targets, which are met with combined flows from the tributary and reservoir releases.

The objective of the numerical example is to maximize water supply for both E-flow requirements and irrigation demands, by using weekly time steps over 22 consecutive weeks of irrigation season. Five selected water sharing policies were modelled using Equation (8), which ranged from equal relative deficits between E-flows and irrigation to prioritizing each type of water use at the expense of the other, along with 75%/25% and 25/75% sharing arrangements for the remaining two scenarios. Tables 2 and 3 provide the input data.

Return flows are set to 20% of consumptive use on irrigation block, while the reservoir releases to the downstream river reach are made to augment tributary flows such that the E-flow targets are met based on water sharing policies with irrigation for each of the five selected water sharing policies. The differences between Scenario 1 and Scenario 2 are based on the inclusion or exclusion of the outflow elevation curve as a model constraint, while both Scenarios 1 and 2 enforce equal relative deficit sharing between E-flows and irrigation.

The purpose of comparing the two scenarios is to demonstrate the differences in optimal solutions with and without the reservoir outflow constraints. The starting storage level for all simulations was 24 m, with full supply at 30 m and dead storage level at 5 m. When the control structure is used (as in Scenario 1), the invert of the outflow structure located at the

Table 1. Percentage of surveyed publications concerning the classification criteria.

Classification	Involvement of water managers as co-authors	The publication provides useable reservoir operating guidelines	Actual use of results reported in the industry
Yes	2.54%	9.14%	2.54%
No	94.92%	75.13%	93.91%
Unclear	2.54%	15.74%	3.55%

Table 2. Reservoir capacity and outflow curve for irrigation canal.

Reservoir elevation (m)	Water surface area (ha)	Volume (1000 m^3)	Reservoir elevation (m)	Qmax diversion (m^3/s)
0	0	0	10	0.00
10	300	15 000	15	5.40
20	400	50 000	20	9.00
30	500	95 000	25	11.34
			30	12.60

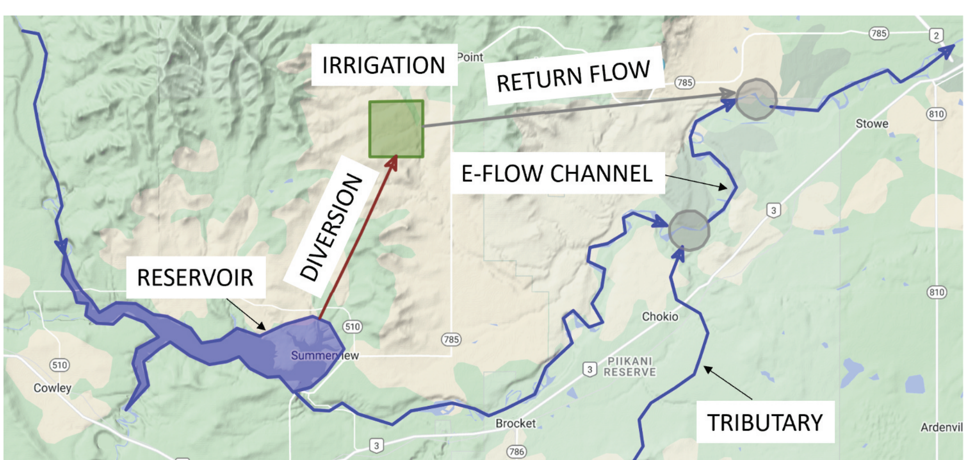

Figure 9. Schematic layout of the numerical example.

Table 3. Reservoir capacity and outflow curve for irrigation canal.

	Input data						Scenario 1		Scenario 2	
Date	Reservoir inflow (m³/s)	Irrigation demand (m³/s)	E-flow target (m³/s)	Tributary inflow (m³/s)	Evap. (mm)	Precip. (mm)	Irrigation supply (m³/s)	E-flow supply (m³/s)	Irrigation supply (m³/s)	E-flow supply (m³/s)
1 April 2023	3.724	2.040	1.676	0.561	7	0	1.545	1.272	1.888	1.555
8 April 2023	3.220	3.091	1.449	0.349	7	0	2.340	1.098	2.860	1.342
15 April 2023	3.080	4.423	1.386	0.371	8	0	3.347	1.052	4.090	1.286
22 April 2023	3.860	5.862	1.737	0.583	10	0	4.437	1.317	5.423	1.610
29 April 2023	4.752	7.273	2.138	0.765	12	0	5.505	1.620	6.728	1.980
6 May 2023	8.122	8.555	3.655	1.539	14	0	6.474	2.771	7.912	3.387
13 May 2023	11.034	9.639	4.965	1.458	16	0	7.299	3.763	8.921	4.599
20 May 2023	13.101	10.482	5.895	1.623	18	0	7.935	4.467	9.698	5.460
27 May 2023	14.144	11.067	6.365	1.563	20	0	8.382	4.816	10.244	5.886
3 June 2023	14.157	11.397	6.371	2.077	22	29	8.632	4.823	10.550	5.895
10 June 2023	13.267	11.489	5.970	1.631	24	43	8.700	4.520	10.633	5.525
17 June 2023	11.700	11.374	5.265	1.370	25	0	8.609	3.983	10.522	4.868
24 June 2023	9.743	11.093	4.384	1.071	27	58	8.397	3.316	10.263	4.053
1 July 2023	7.710	10.690	3.469	0.913	29	60	8.094	2.627	9.893	3.211
8 July 2023	5.901	10.212	2.655	0.723	30	59	7.731	2.014	9.449	2.462
15 July 2023	4.570	9.704	2.057	0.607	32	56	7.344	1.560	8.977	1.906
22 July 2023	3.886	9.203	1.749	0.459	34	0	6.966	1.325	8.514	1.620
29 July 2023	3.897	8.739	1.753	0.426	36	46	6.618	1.325	8.088	1.620
5 August 2023	3.600	8.326	1.620	0.462	38	39	6.307	1.227	7.709	1.499
12 August 2023	3.700	7.962	1.665	0.481	40	0	6.027	1.264	7.366	1.546
19 August 2023	3.400	7.625	1.530	0.517	41	0	5.777	1.158	7.061	1.416
26 August 2023	3.200	7.268	1.440	0.451	42	0	5.505	1.090	6.728	1.333
						Total	85.86	31.70	104.93	38.74

elevation of 10 m becomes *de facto* the dead storage level, since the storage level below the invert results in zero flows into the irrigation canal, and that would also stop water allocation to E-flows in any scenario where there is equal deficit sharing between irrigation and E-flows. It is instructive to observe the difference in optimal reservoir levels between Scenarios 1 and 2, which are both based on equal deficit sharing between E-flows and irrigation, and differ only by the inclusion of the outflow vs elevation curve in the model vs ignoring it, as shown in Fig. 10. Although the water demands for E-flows and irrigation are identical in Scenario 1 and 2, the inclusion of the outflow constraint results in a different solution in Scenario 1, with about 22.2% lower supply compared to Scenario 2 where the outflow vs elevation constraint is included in the model. Figure 11 shows that the resulting optimal rule curves obtained for Scenarios 1 and 2 are quite different, while this difference is only caused by including or excluding the outflow vs elevation curve as an optimization constraint.

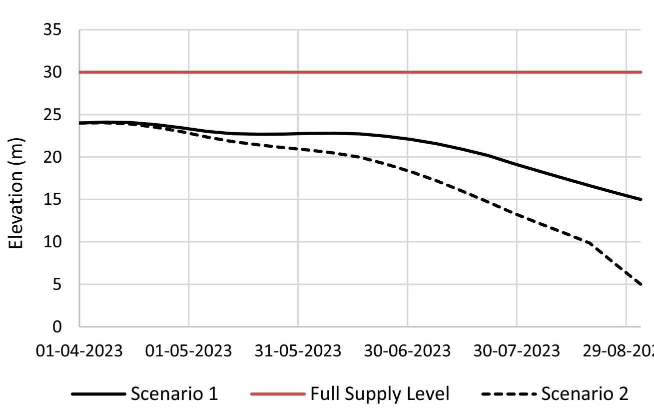

Figure 10. Comparison of optimal reservoir levels for Scenarios 1 and 2.

A total of five scenarios that utilize the outflow vs elevation curve were developed with deficit sharing ranging from the maximum priority given to either irrigation or E-flows on each end of the spectrum, with three additional scenarios that involve sharing of relative deficits either 50/50 or by giving a predetermined amount of higher share to each type of water use at the expense of the other. Each of these five solutions was obtained after less than 1 second of computer run time and they are sufficient to create the Pareto-optimal front shown in Fig. 12 in the units of mean annual water supply to each water use on the left side of the graph, and in terms of the mean relative deficits as a percentage of the target demands.

There are some advantages to obtaining the Pareto front in this way: (a) each of the points in the curve is guaranteed to be globally optimal (i.e. the best possible) subject to the input data and assumed water-sharing policy (as opposed to being "approximate," resulting from the use of heuristic solvers); (b) each of the solutions is guaranteed to comply with the reservoir elevation vs outflow constraints and the deficit sharing constraints; and, (c) the computational effort of getting the Pareto-optimal front is smaller by several orders of magnitude compared to the use of heuristic solvers, which enables the use of this approach on much larger problems with shorter time steps that may also include hydrological routing (Ilich and Basistha 2024). All of these advantages result from the nature of LP, which seems to be downplayed in many recent publications as an example of "outdated" technology. The test problem presented here may seem trivial, but most heuristic solvers may have significant difficulties finding the solutions to Scenario 1 shown in Table 2 due to the complexities of the constraints that need to be enforced over 22 weeks simultaneously between two diverse components, one of which also has the local runoff contribution. A proper account of net evaporation has also been

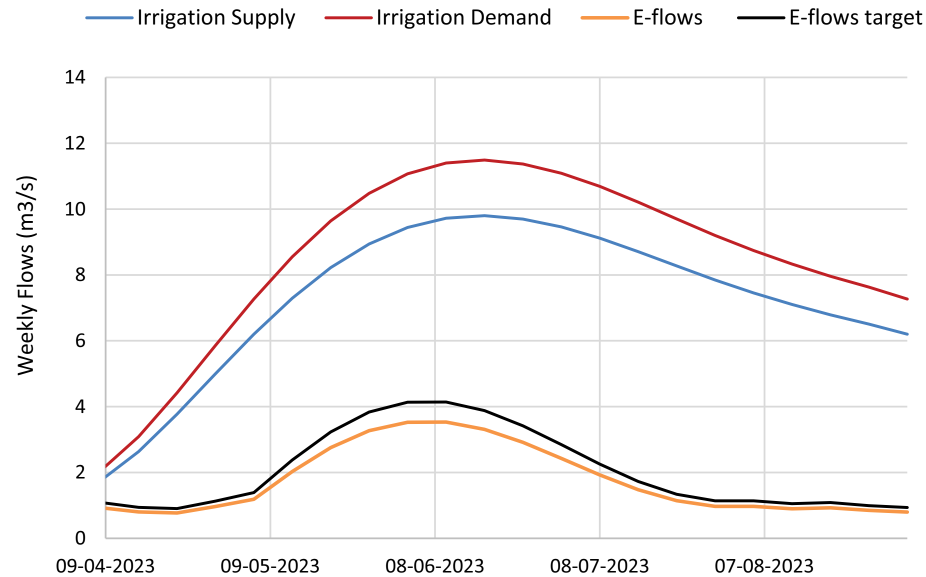

Figure 11. Irrigation and E-flow water use and target demands in Scenario 1.

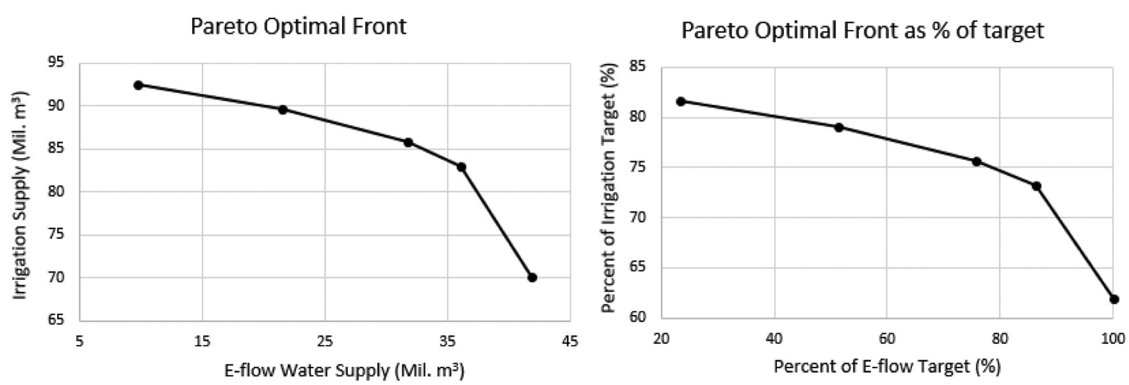

Figure 12. Pareto-optimal front expressed in the units of volume and percent deficit.

included in the simultaneous optimization of reservoir operation for 22 consecutive weeks in this example.

6 Conclusions and recommendations

This paper provides a targeted review of the current state of the art in river basin modelling, with an emphasis on the applicability of model solutions for basin managers and reservoir operators. The results of this survey indicate that there is still a significant disconnect between academia and practitioners. The main findings can be summarized through the following points:

- Most planning studies aiming to develop reservoir rule curves rely on the use of monthly calculation time steps. This causes gross simplification of inflow hydrographs, with negative effects on the accuracy of the model results that have yet to be properly evaluated and reported in the literature.
- Most modelling studies ignore the effect of reservoir outflow constraints and deliver solutions of questionable quality since they may not be physically possible during low storage levels.
- Net evaporation is not modelled properly in a large majority of modelling studies. It is either modelled as evaporation (without its precipitation component), or it is ignored altogether.
- Only a handful of optimization models that were surveyed display the ability to be used as real-time operational tools, mainly due to their inability to handle channel routing that is required for daily or sub-daily reservoir release decisions in multi-reservoir systems. In addition to the low reliability of real-time runoff forecasts, the lack of ability to address these processes in most models with sufficient accuracy is probably one of the principal reasons for the gap between theory and practice when it comes to applying the model results as guidance for reservoir releases in real-time.
- Multi-objective optimization has introduced additional complexity into modelling by generating a multitude of solutions without clear guidance to practitioners on how to apply these solutions in their day-to-day decision

making to improve the management of reservoir releases. Clear guidelines on how the problems should be formulated and solved are missing, resulting in a multitude of ideas among researchers that have achieved no traction among practitioners. Also, the above remarks on the importance of proper modelling of constraints are also applicable to multi-objective optimization.
- LP solution algorithms are still the premier tools for river basin management models, capable of solving large problems with complex constraints much faster than any other solution methodologies, while simultaneously guaranteeing finding solutions that are globally optimal. Many researchers seem to have lost sight of this fact.

The surveyed papers document various attempts that have been made to infer reservoir operating rules from the results of river basin optimization models. The findings on the nature of practicality of the reservoir operating rules are as follows:

- The use of regression (linear or non-linear) that relates the starting storage level and anticipated (forecasted) reservoir inflow for a given time step to the reservoir outflows remains one of the common techniques for the generation of the reservoir release rules. This approach has recently been complemented by using ANN and other machine learning algorithms instead of regression. However, typical difficulties are the use of monthly or 10-daily time steps in these studies, which would require reliable monthly or 10-daily runoff forecasts for real-time applications. The other disadvantage of this method is that it derives reservoir releases without taking into account the current downstream water demands that may be partially met by downstream runoff and downstream rainfall, the effects of which are typically ignored by regression models.
- Reservoir rule curves have been defined in most of the surveyed studies as the storage trajectories obtained from optimization models over a range of inflow conditions. More refined studies resulted in three different rule curves, corresponding to typical dry, median or wet years. However, these rule curves are still fixed, while they should depend on the starting storage levels inherited from previous years and the runoff/demand conditions in each year, and they may differ significantly from year to year. A universally acceptable method of analysing numerous optimal reservoir trajectories to create practical guidelines for reservoir operation in typical dry, median and wet years is still missing.
- Some proposals for joint management of storage using reservoir operating zones that are related in varying degrees of water rationing policies seem to hold promise for improved drought management that is relatively easy to follow and implement. These proposals do not rely on inflow forecasts, but they do take into account the time of the year and starting storage levels.
- Similarly, the model predictive control (MPC) approach seems to be very promising for reservoir management during floods. The difficulty is the need to provide real-time data from the field (starting river and storage conditions), along with a reliable short-term runoff forecast. Given the massive efforts in the development of sophisticated weather forecasts, improved remote sensing and the use of AI to improve rainfall-runoff models, the eventual synergy between these emerging technologies and the MPC approach may yet be the best option for computerized assistance of future real-time reservoir operation during floods.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This study has been jointly funded by Alberta Innovates, the Ministry of Environment and the Ministry of Agriculture and Irrigation, Alberta, Canada.

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